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A NEW CORRECTION TO EDM SLOPE DISTANCES IN APPLICATIONS OF UNTILTED CORNER REFLECTORS, EFFECTS OF MISALIGNED REFLECTORS AND DETERMINATION OF REFLECTOR CONSTANTS

Final Technical Report

L. A. Kivioja and William A. Oren

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A NEW CORRECTION TO EDM SLOPE DISTANCES IN APPLICATIONS OF UNTILTED CORNER REFLECTORS, EFFECTS OF MISALIGNED REFLECTORS AND DETERMINATION OF REFLECTOR CONSTANTS

FINAL TECHNICAL REPORT

by

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TABLE OF CONTENTS

ABSTRACT

Errors caused by single and multiple reflectors in Electronic Distance Measuring (EDM) are studied, and methods for their eliminations are given. Formulas by which these errors can be computed are derived, and practical numerical examples are given and solved. The results are also verified by actual EDM measurements using the Kern ME-3000 Mekometer. The described methods are applicable for all types and makes of EDM reflectors, and for all feasible applications of these reflectors. Errors produced by any misalignment in azimuth and/or vertical can be computed by these formulas. A common misalignment occurs when nontiltable reflectors are used over slope distances because most measured lines have some slope. These errors can be easily corrected, even for historical measurements, because the elevations or the slopes are usually known. Reflector systems consisting of multiple reflectors must be aligned with precision so that the reflectors on the opposite sides of the plumbing point are at the same distance from the EDM instrument, if not, large errors are easily introduced into the measured distances. For this reason, one large reflector produces more accurate results than several (usually three) smaller reflectors having the same reflective area. This report also describes a precise method for determining the index of refraction for any corner reflector by using standard surveying instruments. When the index of refraction of the reflector is known together with its size, and with its location over the plumbing point, almost any reflector can be used with almost any EDM instrument. Whenever centimeter accuracy -or better- is desired for an EDM distance, errors caused by the reflectors must be either avoided, or they must be computed by the methods described in this research. Only a few tiltable reflectors are available which produce no errors when they are correctly placed and aligned over the end point of the measured EDM line.

INTRODUCTION

In common surveying practice the reflectors used in electronic distance measuring (EDM) are leveled and positioned so that their frontal planes are vertical facing the EDM instrument. Only a few reflectors are tilted correctly. Whenever the horizontal and vertical alignments are not perfect, or when slope distances are measured using untiltable reflectors, the incoming light or infrared radiation will not hit the frontal planes at 90^0 angles. The optical path inside the corner reflector is minimum when the light hits the frontal plane at a 90^0 angle, while greater values are obtained for other incoming angles.

The optical path to the EDM instrument will therefore change when the reflector is turned in azimuth, or vertical. This change depends on the physical locations of the vertical and horizontal axes about which the reflector can be rotated to point to the EDM instrument.

In addition to the locations of the axes, the plumbing point, which is the end point of the line to be measured by the EDM instrument, has various locations within or outside the reflector depending on the reflector mount, and how it fits on the tripod.

The reflector"constant" is, of course, also a function of the thickness and the index of refraction of the corner reflector. For example, if the distance from the frontal plane of a glass corner reflector to its apex is 6.6 cm, and if the index of refraction of the glass is 1.57, and if the incoming light hits the frontal plane 30^0 off its normal, the optical path is 0.36 cm greater than in the normal (90^0) case. Wherever corner reflectors are used and centimeter accuracies are desired, the described effects should be considered. All reflectors can be rotated in azimuth, and some can be tilted, but the point on the ground to which the distance is obtained may not be the desired point depending on the geometric locations of the vertical and horizontal axes of rotation.

The dimensions associated with the reflectors can best be seen in Figs. 11 and 12 on pages 33 and 34. These figures should be referred to as needed when studying the derivations of the formulas for various reflectors.

1. OPTICAL PATH INSIDE A SOLID CORNER REFLECTOR

By definition, the optical path inside a medium with an index of refraction n over a distance s is ns. In Fig. 1, a corner reflector having an index of refraction n, receives light, or infra-red radiation to its

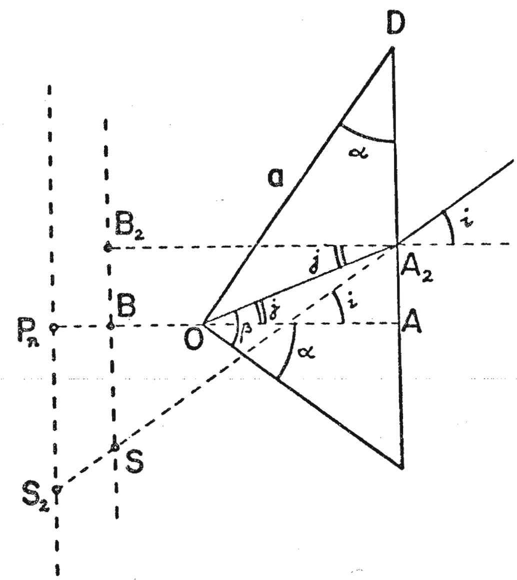


Figure 1. Ray to the apex in a corner reflector and the geometry associated with slope distances.

frontal plane DA at a 90^o angle parallel to line OA. The length of the optical path of all these parallel rays inside the reflector is n*OA. When the incoming light enters the reflector with inclination i, such as the ray entering at A_2, the optical path inside the reflector is $n*OA_2$, [1].

The reflector constant is a function of the incident angle, the size of the corner reflector, its index of refraction, and, of course, on how the reflector is mounted in its housing with regard to the plumbing point.

The value of $n{*}OA_2$ is given by the following equation;

$$n{*}OA_2 = \frac{n^2 a}{\sqrt{3}\,\sqrt{n^2 - \sin^2 i}} = \frac{n^2 d}{\sqrt{n^2 - \sin^2 i}} \qquad (1)$$

where n = index of refraction of the reflector

a = OD = length of the edge of the corner cube

i = angle of incidence to the frontal plane, Fig. 1.

$d = OA = \frac{a}{\sqrt{3}}$ = thickness of the reflector

This Eq. (1), gives the optical path length inside the corner reflector for all inclinations i. The formula is derived in [1].

2. CORRECTION TO SLOPE AND HORIZONTAL DISTANCES WHEN USING AN UNTILTED CORNER REFLECTOR

If a corner reflector such as the one in Fig. 1 is correctly used for horizontal distances, the distance is actually obtained to point P_n. The measured distance is from the EDM instrument to point A, plus the optical path inside the reflector which is:

$$AP_n = n \times OA = \frac{na}{\sqrt{3}} = nd \qquad (2)$$

The physical location of the plumbing point and the reflector "constant" do take this into account giving the desired distance.

When this same corner reflector is used for slope distances with inclinations $\pm$ i to the vertical frontal surface DA, the slope distance is measured to point S in Fig. 1 where

$$SA_2 = n \times OA_2 = \frac{n^2 a}{\sqrt{3}\sqrt{n^2 - \sin^2 i}} = \frac{n^2 d}{\sqrt{n^2 - \sin^2 i}} \qquad (1)$$

Note that point S is not on the vertical through point P_n. The error in slope distance is SS_2 which is

$$\Delta s_i = SS_2 = S_2A_2 - SA_2 = \frac{AP_n}{\cos i} - SA_2 ,$$

or

$$\Delta s_i = SS_2 = \frac{nd}{\cos i} - \frac{n^2 d}{\sqrt{n^2 - \sin^2 i}} ,$$

or

$$\Delta s_i = SS_2 = nd\left(\frac{1}{\cos i} - \frac{n}{\sqrt{n^2 - \sin^2 i}}\right) . \qquad (3)$$

The correction to horizontal distances is

$$\Delta s_h = SS_2 \times \cos i = BP_n$$

$$\Delta s_h = BP_n = nd\left(1 - \frac{n \cos i}{\sqrt{n^2 - \sin^2 i}}\right) = nd - \frac{n^2 d \cos i}{\sqrt{n^2 - \sin^2 i}} \qquad (4)$$

For untilted corner reflectors, equation (3) gives the desired correction to slope distances, and equation (4) gives the desired correction to horizontal distances. These corrections are proportional to the product nd, and therefore small corner reflectors, such as plastic reflectors for which d is small, have small corrections, and of course, large corner reflectors have large corrections. Note that, for any one reflector, these corrections do not depend on how long the measured line is. They depend only on n, d, and the angle of incidence $\pm$ i which is obtained either by measuring the vertical angle from the EDM instrument to the apex of the reflector, or by computing the same from elevation differences. The measured slope distance is "too short" by $S_2S = \Delta s_i$. This correction should be added to the measured slope distance. If the reflector is tilted or turned, one can compute the required correction from the geometry of each observational situation. Note that the optical path inside the corner reflector always increases with increasing angle of incidence.

It is not uncommon for a land surveyor to measure up or down a 30^o slope using an untilted corner reflector, and if the corrections given by equations (3) and (4) are ignored, the results will be in error by several millimeters.

Formulas (3) and (4) are for "zero off-set" plumbing points, meaning that the reflector in its housing is so situated that its points P_n and S_2 in Fig. 1 are on the plumbline of the end point of the measured EDM line. Some commercially available reflectors are of this type. This means that in Fig. 1 the actual optical path for a horizontal distance is displayed by a zero off-set EDM instrument to point P_n, where point P_n is on the plumbline of the plumbing point. Exactly the same distance would be obtained if the corner reflector in Fig. 1 would be removed and a vertical front surface mirror would be placed through points P_n and S_2. Other physical locations for the plumbing point are in use with various corner reflectors.

If the plumbing point in Fig. 1 is moved to the right by Δe, and if the reflector remains stationary, the EDM instrument will continue to display the original optical path for a horizontal distance to point P_n. However, the desired distance is shorter than the actual optical path by Δe. If the display on the EDM instrument is off-set by minus Δe from the actual optical path, the desired distance will be displayed. After such an off-set, this instrument will subtract Δe from all distances including the slope distances. For slope distances, the horizontal component of this off-set is $\Delta e \cos i$, and equation (4) then becomes

$$\Delta s_h = nd\left(1 - \frac{n \cos i}{\sqrt{n^2 - \sin^2 i}}\right) - \Delta e(1 - \cos i) \tag{5}$$

One can see that by choosing Δe properly, one can make Δs_h equal to zero for any desired angle i, or one can reduce Δs_h, or one can even make Δs_h negative. Various corner reflectors have values for Δe ranging between zero and the thickness d of the corner reflector depending on the location of the plumbing point.

3. ROTATIONS OF SINGLE REFLECTORS IN AZIMUTH AND IN VERTICAL

3.1 Axis of Rotation is Inside the Reflector

The equation for the optical path length difference (PLD) of a single reflector which is misaligned by an angle α and whose axis of rotation, C, is "within" the reflector, is derived first. Misalignment α may be in azimuth or in vertical. From Fig. 2, it can be seen that the optical path taken by the incoming light ray when the reflector is perpendicular to the ray is:

$$D = s + nd \tag{6}$$

where s is the optical path from the EDM instrument to point A_{10} on the front surface of the reflector before the rotation, $d = OA_{10} = OA$, is the thickness, and n is the index of refraction of the reflector.

When the reflector is rotated around axis C by an angle α, its new position $O_2G_2E_2$ is shown by the dotted outline in Figure 2. The optical path of the incoming light will then be: $s - A_2F + n*O_2A_2$, or:

$$D_2 = s + \frac{nd}{\cos\beta} - A_2F \tag{7}$$

In this equation s and d are the same as above and the angle of refraction β can be found by Snell's law: $n = \sin\alpha/\sin\beta$ when the angle of misalignment α is known. A_2F, whose value is given by Eq. (9) is a perpendicular dropped

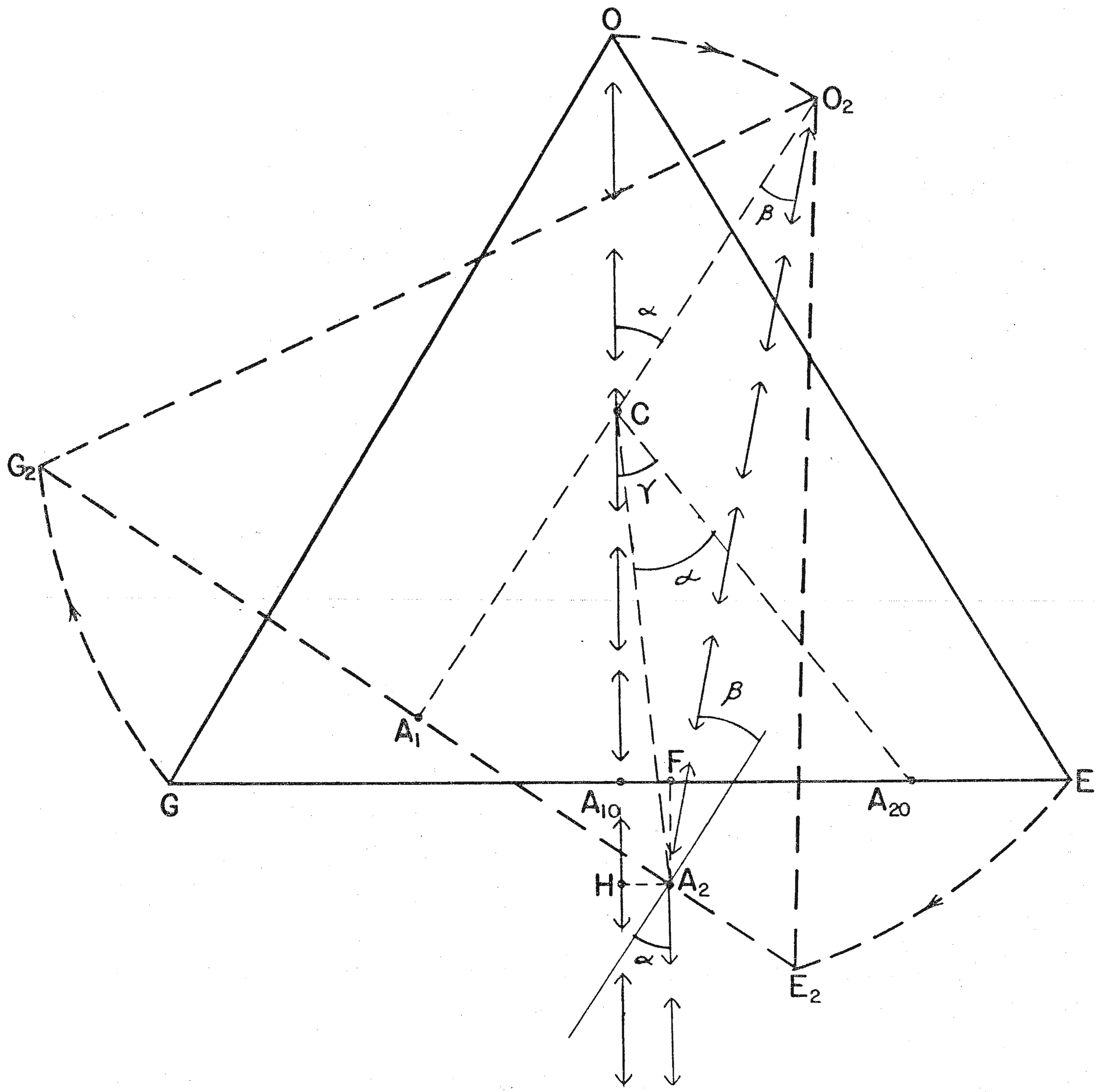


Figure 2. Single reflector OGE with its axis of rotation C inside the reflector is rotated by an angle $\alpha = \sphericalangle OCO_2$ to position $O_2G_2E_2$. $CA_{10} = r$. Ray going to the apex enters the reflector at A_{10} before the rotation, and at A_2 after the rotation.

from the reflector frontal plane at its initial position to point A_2 where light is incident upon the misaligned reflector. From the right ΔCA_1A_2, one sees that

$$CA_2 = \sqrt{(d \tan \beta)^2 + r^2} \tag{8}$$

where $r = CA_{10} = CA_1$ is the measured distance from the axis of rotation to the front of the reflector and $d \tan \beta = A_1A_2 = A_{10}A_{20}$. Having found CA_2, one can find A_2F as follows:

$$A_2F = A_{10}H = CA_2 \cos (\gamma - \alpha) - r \tag{9}$$

where

$$\gamma = \tan^{-1}(\frac{d\tan\beta}{r}) \tag{10}$$

It can be seen from Fig. 2 that $A_2F = A_{10}H$.

Subtracting Eq. (6) from Eq. (7) one obtains the path length difference PLD:

$$PLD = nd(\frac{1}{\cos\beta} - 1) - A_2F \tag{11}$$

Applying equations (8) and (9), the final form for the optical path length difference is:

$$PLD = nd\left(\frac{1}{\cos\beta} - 1\right) + r - \cos(\gamma - \alpha)\sqrt{(d\tan\beta)^2 + r^2} \tag{12}$$

By using equation (12) one can find the path length difference resulting from misalignment of a reflector by an angle α. To do this n, d, α and r must be known. The variables d and r must be measured for each reflector and angle α for every misalignment. The index of refraction n may also have to be determined if its value is not known. Both horizontal and vertical length differences can be computed by Eq. (12) if r is the distance from the vertical or horizontal axis of rotation.

3.2 Axis of Rotation is in Front of the Reflector

Another situation is encountered when the axis of rotation is in front of the reflector. This is shown in Figs. 3a and 3b. The equation to calculate the path length difference for this case will be derived next.

As in the previous case, the distance measured before rotation is found through Eq. (6) and after rotation by Eq. (7). Eq. (8) also is valid for this case, however, it can be seen from Figs. 3a and 3b that Eq. (9) will change. Instead of Eq. (9) one has:

$$A_2F = A_{10}H = r - CA_2 \cos(\alpha + \gamma) \tag{13}$$

where $r = CA_{10}$ is the distance from the axis of rotation at C to the front of the reflector and CA_2 and γ are computed using Eqs. (8) and (10) respectively. It can be seen that $A_2F = A_{10}H$. The path length difference PLD is then computed as before in Eq. (11). After applying Eqs. (8) and (13) the final form is:

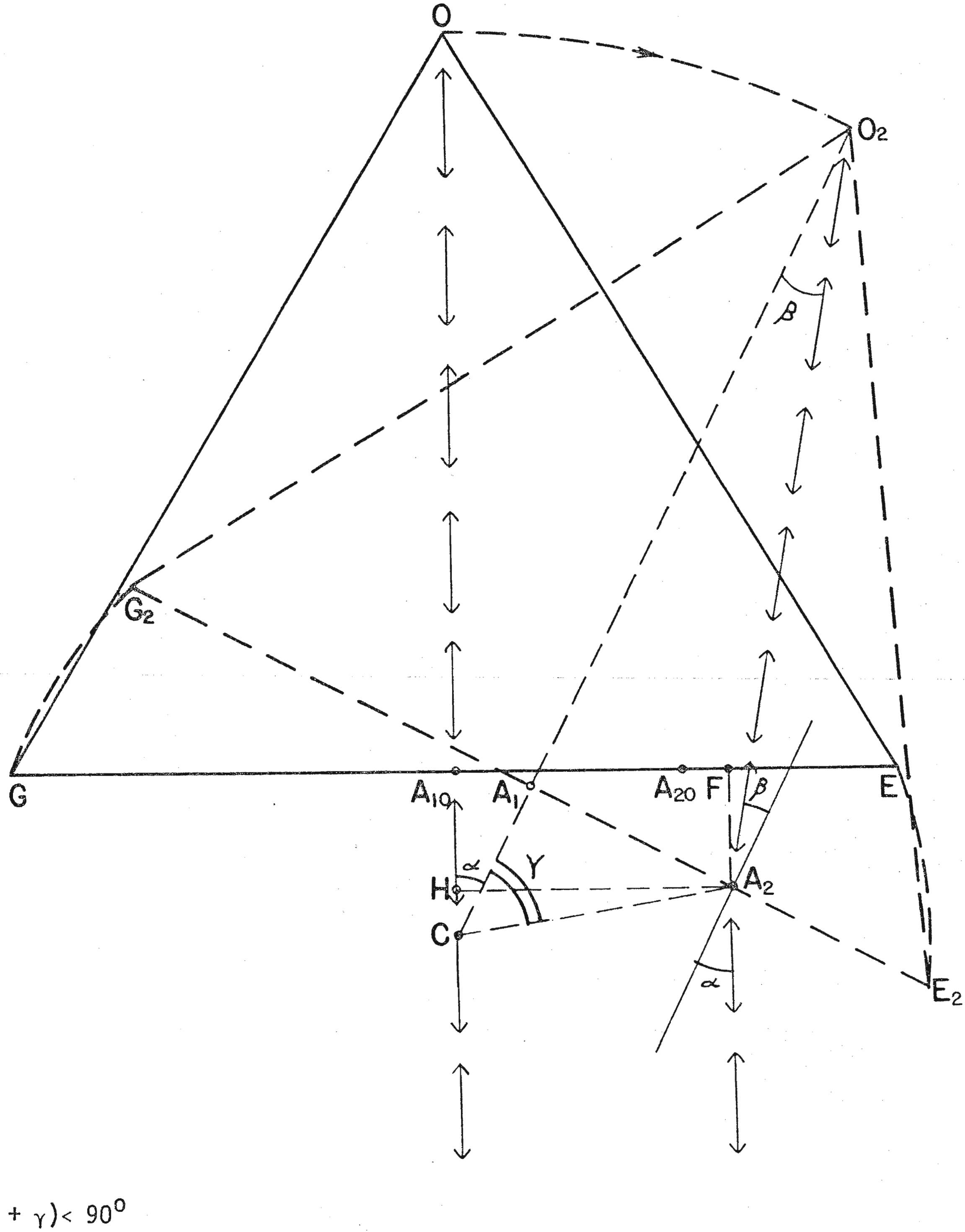
$$PLD = nd\left(\frac{1}{\cos\beta} - 1\right) - r + \cos(\alpha + \gamma)\sqrt{(d \tan\beta)^2 + r^2} \tag{14}$$

Again by knowing d, n, and r for the particular reflector, one can compute γ and β, and solve Eq. (14) for any angle of misalignment α.

3.3 Axis of Rotation is Behind the Reflector

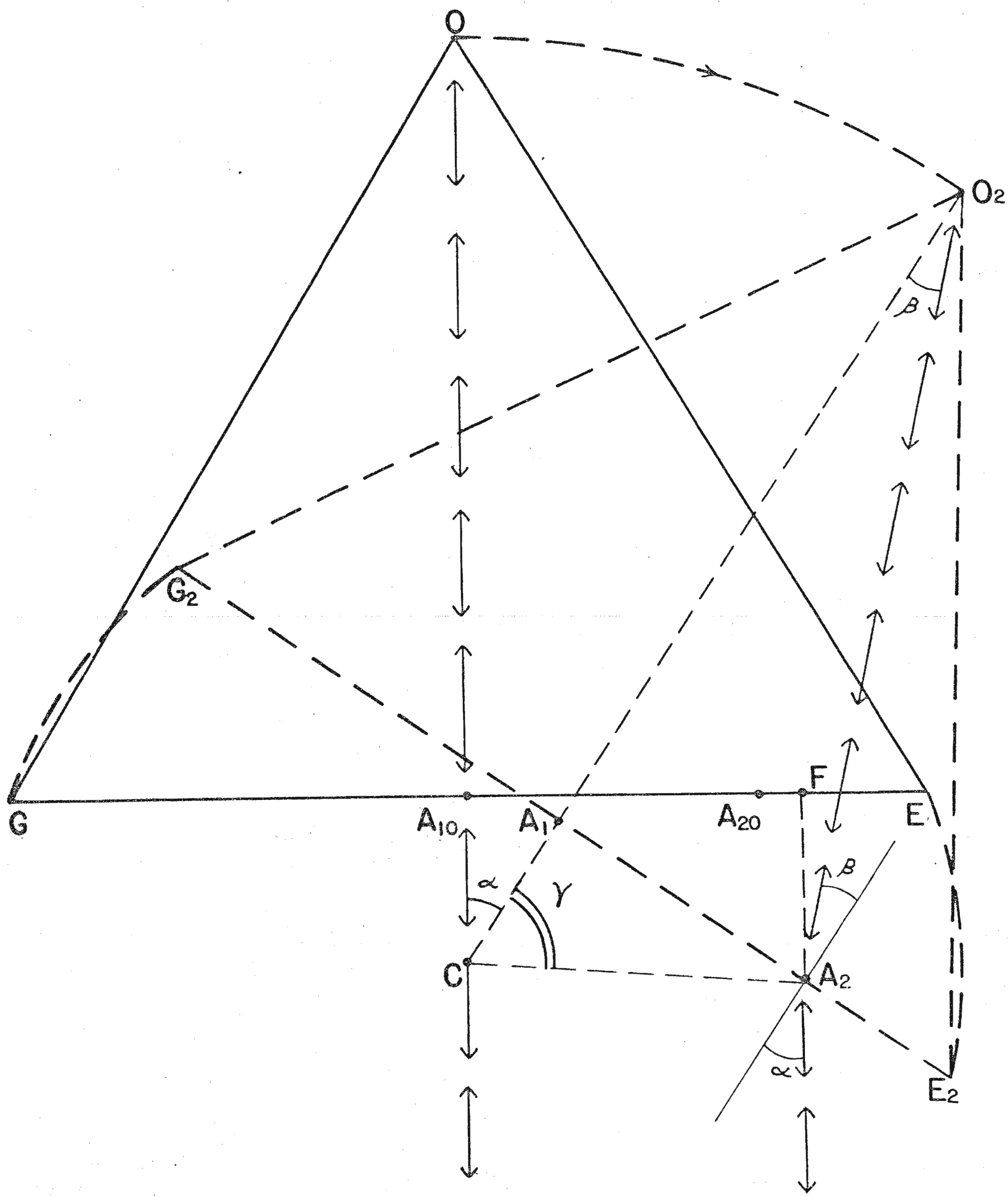
The third case involves computing the optical path length difference PLD for a misaligned corner reflector with its axis of rotation behind the apex, see Fig. 4. Again the distance measured before rotation is computed by Eq. (6) and after rotation by Eq. (7). Eq. (8) remains the same, but as can be seen from Fig. 4, Eq. (9) changes as follows:

$$A_2F = A_{10}H = r - CA_2 \cos(\alpha - \gamma) \tag{15}$$



$(\alpha + \gamma) < 90^0$

Figure 3a. Single reflector with its axis of rotation C in front of the reflector is rotated by an angle α. $CA_{10} = r$. Ray to apex enters at A_{10} before the rotation and at A_2 after the rotation.



$(\alpha + \gamma) > 90^{0}$

Figure 3b. Single reflector with its axis of rotation C in front of the reflector is rotated by an angle α. $CA_{10} = r$. Ray to apex enters at A_{10} before and at A_2 after the rotation.

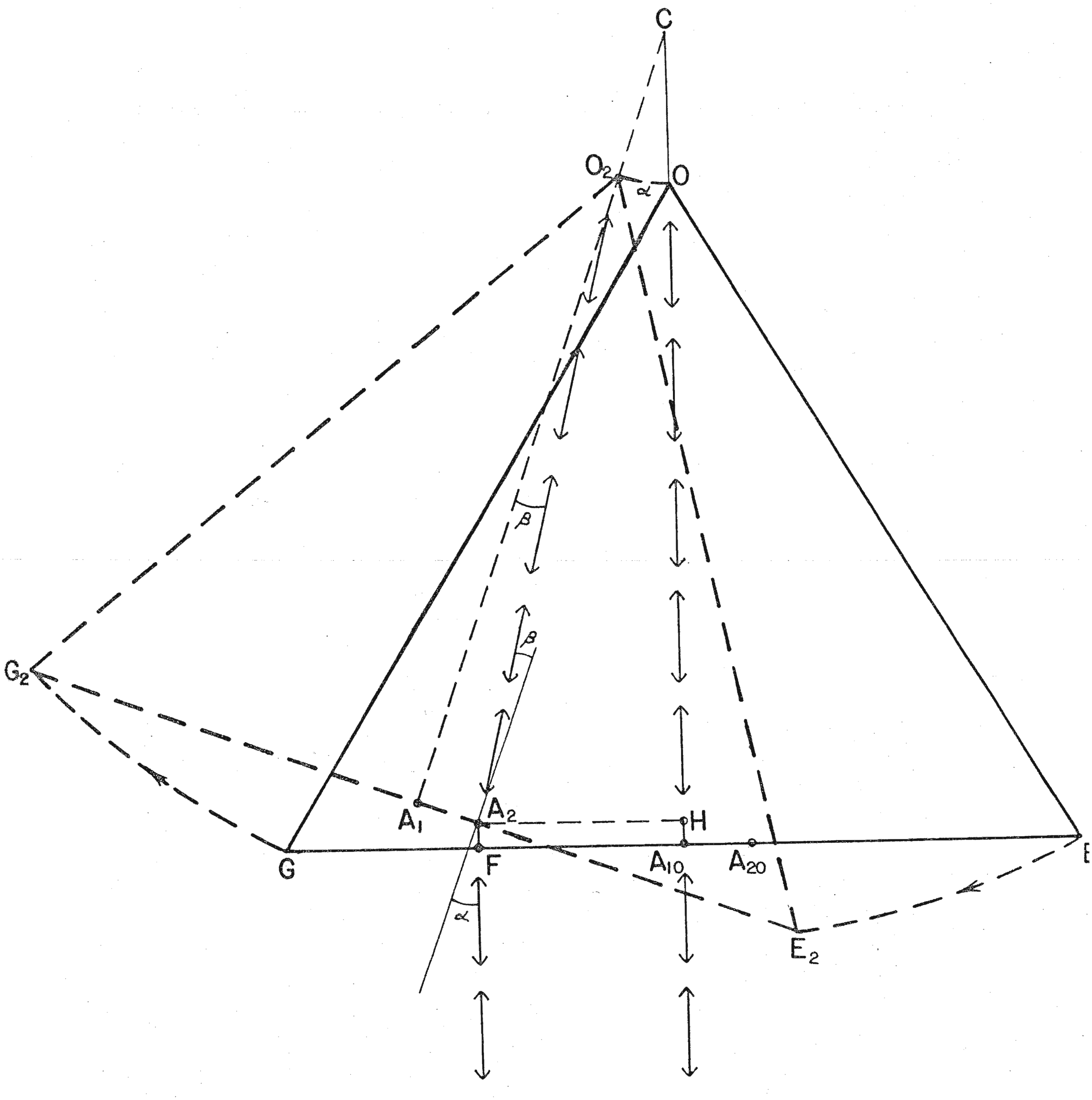


Figure 4. Single reflector with its axis of rotation C behind the apex O, is rotated by an angle α. $CA_{10} = r$. Ray to apex enters at A_{10} and at A_2.

Here CA_2 and $\gamma = \measuredangle A_{10}CA_{20}$ are found as before by Eqs. (8) and (10). The path length difference PLD is then computed using the appropriate equations together with equation (15), resulting in the following:

$$PLD = nd\left(\frac{1}{\cos\beta} - 1\right) + r - \cos(\alpha - \gamma)\sqrt{(d\ \tan\beta)^2 + r^2} \qquad (16)$$

This equation is solved as previously explained.

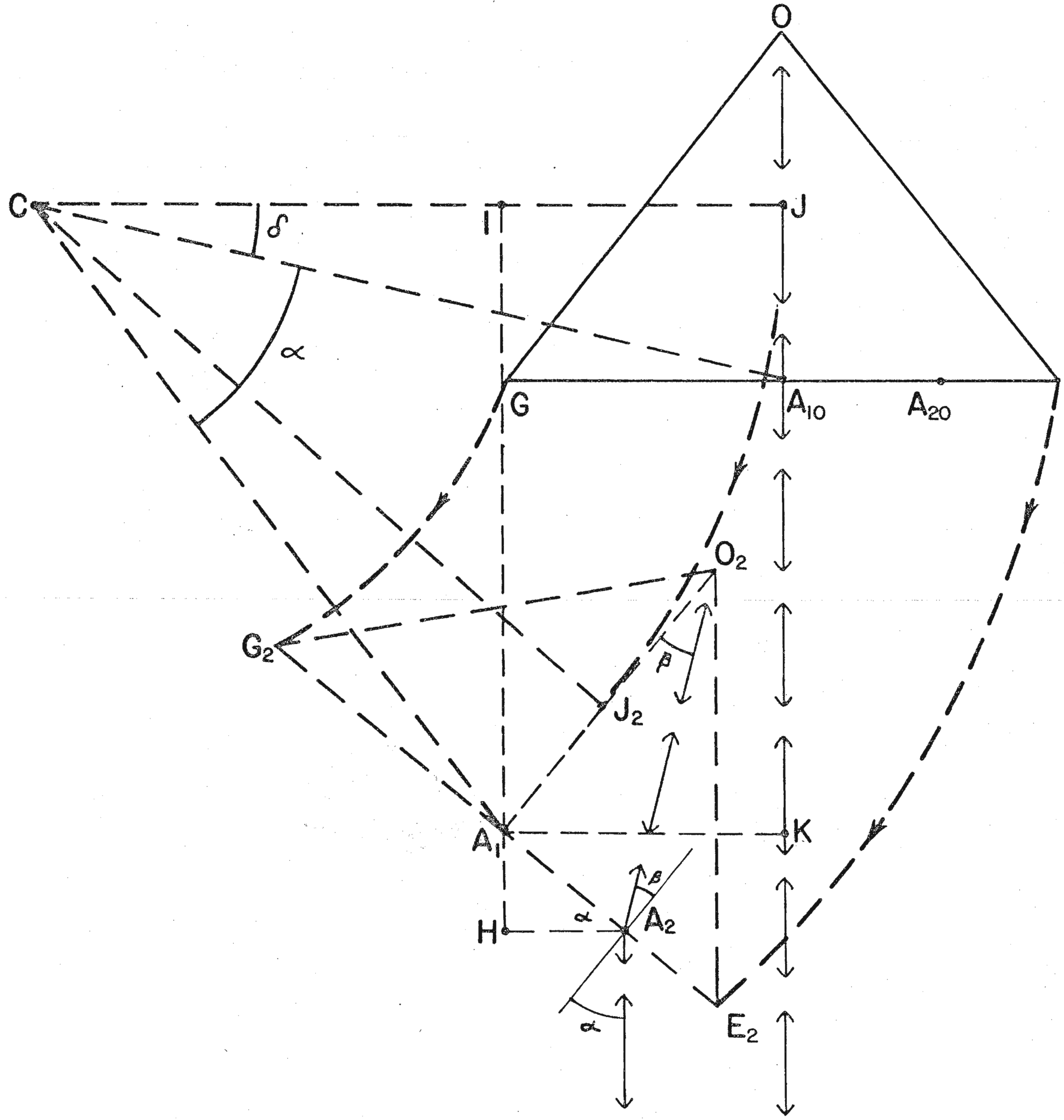


Figure 5. Outer reflector of a triple reflector with axis of rotation C, is rotated by an angle α. $r = JA_{10}$, $e_H = CJ$, $CJ \perp OA_{10}$, and point J is between point O and A_{10}.

4. ROTATIONS OF MULTIPLE REFLECTORS IN AZIMUTH AND IN VERTICAL

4.1 Axis of Rotation is Outside and Beside the Reflector

The derivation of the equation for the path length difference PLD for a triple and/or multiple corner cube reflector with its axis of rotation outside and beside the reflector will be derived next. The situation can be seen in Fig. 5. Four quantities of the reflector must be measured. The first is $r = JA_{10}$, the perpendicular distance from the front face of the reflector to the axis of rotation for that prism. The second is d, the thickness of the reflector which is $A_{10}O$ in the Fig. 5. The third is $e_H = CJ$, the distance from the axis of rotation, to the central vertical plane of the reflector. Finally, n the index of refraction of the reflector, must be known. A method by which n can be obtained will be presented later in this report.

When these four quantities are known, one can find the path length difference PLD. By noting the positions of points O and O_2 in Fig. 5, one can see that small misalignments in angle α will cause large errors. As in the previous cases, the path length before the rotation of the reflector about its axis C is found by using Eq. (6). From Fig. 5, the path length after the rotation is:

$$D_2 = s - A_{10}K - A_1H + \frac{nd}{\cos\beta} , \qquad (17)$$

and also, the length of line $A_{10}K$ is:

$$A_{10}K = IA_1 - r \tag{18}$$

where

$$IA_1 = CA_{10}\sin(\alpha + \delta) \tag{19}$$

where $CA_{10} = CA_1$ and

$$CA_1 = CA_{10} = \sqrt{r^2 + e_H^2} \tag{20}$$

and

$$\delta = \tan^{-1}\left(\frac{r}{e_H}\right). \tag{21}$$

Angle α of misalignment or rotation must be measured for each case. A_1H must still be found. From Fig. 5, one can see from triangles $A_1A_2O_2$ and A_1A_2H that:

$$A_1H = d\tan\beta\sin\alpha \tag{22}$$

where β is the angle of refraction found through Snell's Law. Subtracting Eq. (6) from Eq. (17) one finds the path length difference PLD:

$$PLD = \frac{nd}{\cos\beta} - nd - A_{10}K - A_1H \tag{23}$$

Applying Eqs. (18), (19), (20), (21) and (22) to Eq. (23), one obtains:

$$PLD = nd\left(\frac{1}{\cos\beta} - 1\right) - \sin\left(\alpha + \tan^{-1}\left(\frac{r}{e_H}\right)\right)\sqrt{r^2 + e_H^2} + r - d\tan\beta\sin\alpha \tag{24}$$

Equation (24) enables one to calculate the PLD in both vertical and azimuthal directions of misalignment for triple and/or multiple reflectors with their axes of rotation outside and beside the reflector. Note that the reflectors on one side come closer, and reflectors on the opposite side move farther from the EDM instrument by large amounts. In essence, the EDM instrument simultaneously receives signals from several distances.

4.2 Axis of Rotation is Outside and Ahead of the Reflector

The second case for triple and/or multiple reflectors is when the axis of rotation is outside and ahead of the reflectors, as in Fig. 6. The derivation of the equation for the path length difference is similar to the previous case 4.1.

Referring to Fig. 6, it can be seen that the lengths of the optical paths before and after the rotation are found through Eqs. (6) and (17) respectively. The calculation of $A_{10}K$ by Eq. (18) changes to:

$$A_{10}K = IA_1 + r \tag{25}$$

and equation (19) becomes:

$$IA_1 = CA_{10}\sin(\alpha - \delta) \tag{26}$$

Eqs. (20), (21) and (22) remain the same. Applying the appropriate equations as before, one obtains the equation for the path length difference PLD as:

$$PLD = nd\left(\frac{1}{\cos\beta} - 1\right) - \sin\left(\alpha - \tan^{-1}\left(\frac{r}{e_H}\right)\right)\sqrt{r^2 + e_H^2} - r - d\tan\beta\sin\alpha \tag{27}$$

4.3 Axis of Rotation is Outside and Behind the Reflector

The third situation has the axis of rotation behind the reflectors, as in Fig. 7. The development of the equation to calculate the path length difference is presented next.

It can be seen from Fig. 7 that all the equations used in case 4.1 for triple and/or multiple reflectors (axis inside the reflector) apply here. The path length difference PLD can be computed from Eq. (24) which is repeated below:

$$PLD = nd\left(\frac{1}{\cos\beta} - 1\right) - \sin\left(\alpha + \tan^{-1}\left(\frac{r}{e_H}\right)\right)\sqrt{r^2 + e_H^2} + r - d\tan\beta\sin\alpha \tag{24}$$

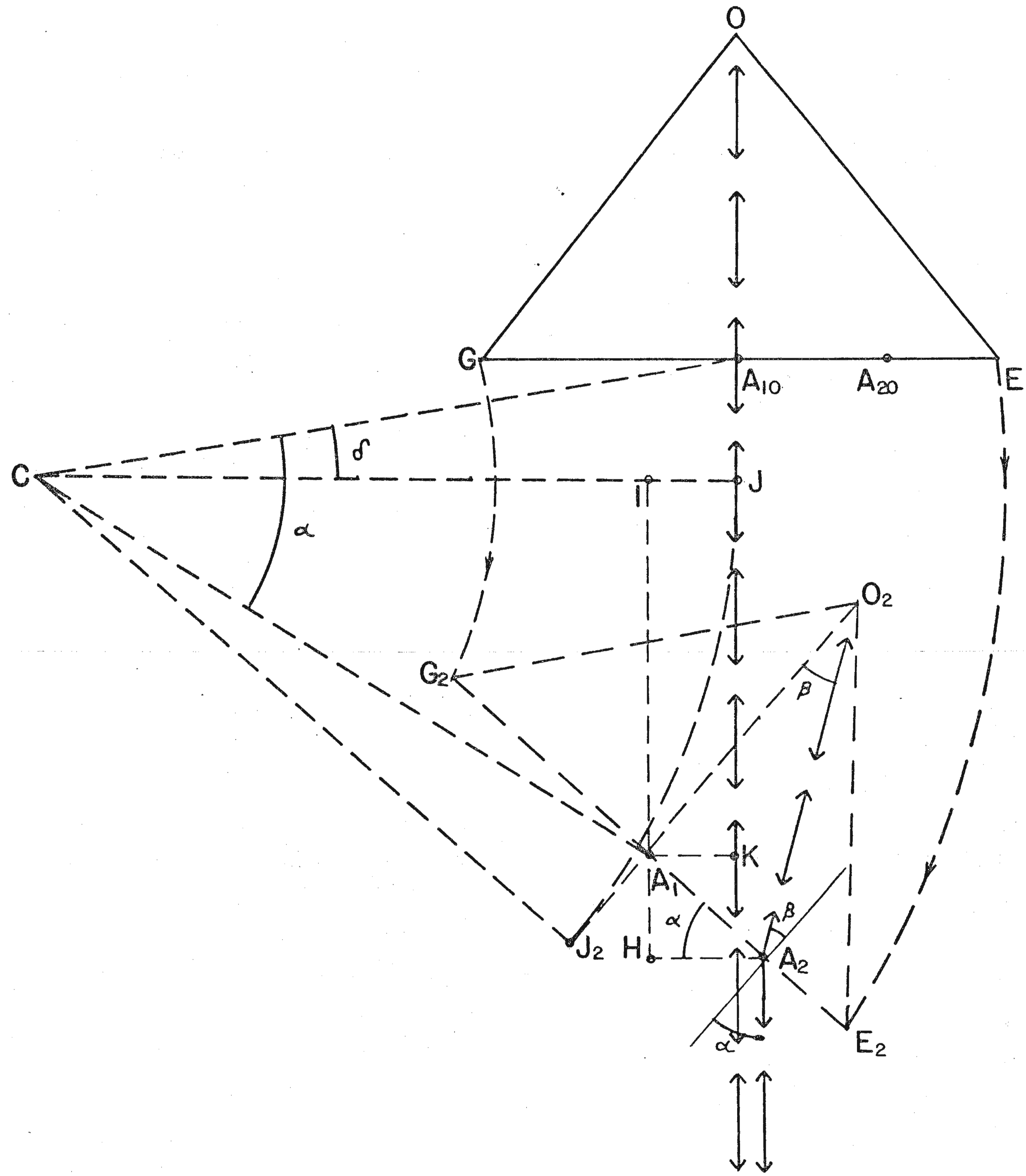


Figure 6. Outer reflector with axis of rotation C in front is rotated by an angle α, with $r = JA_{10}$ and $e_H = CJ$.

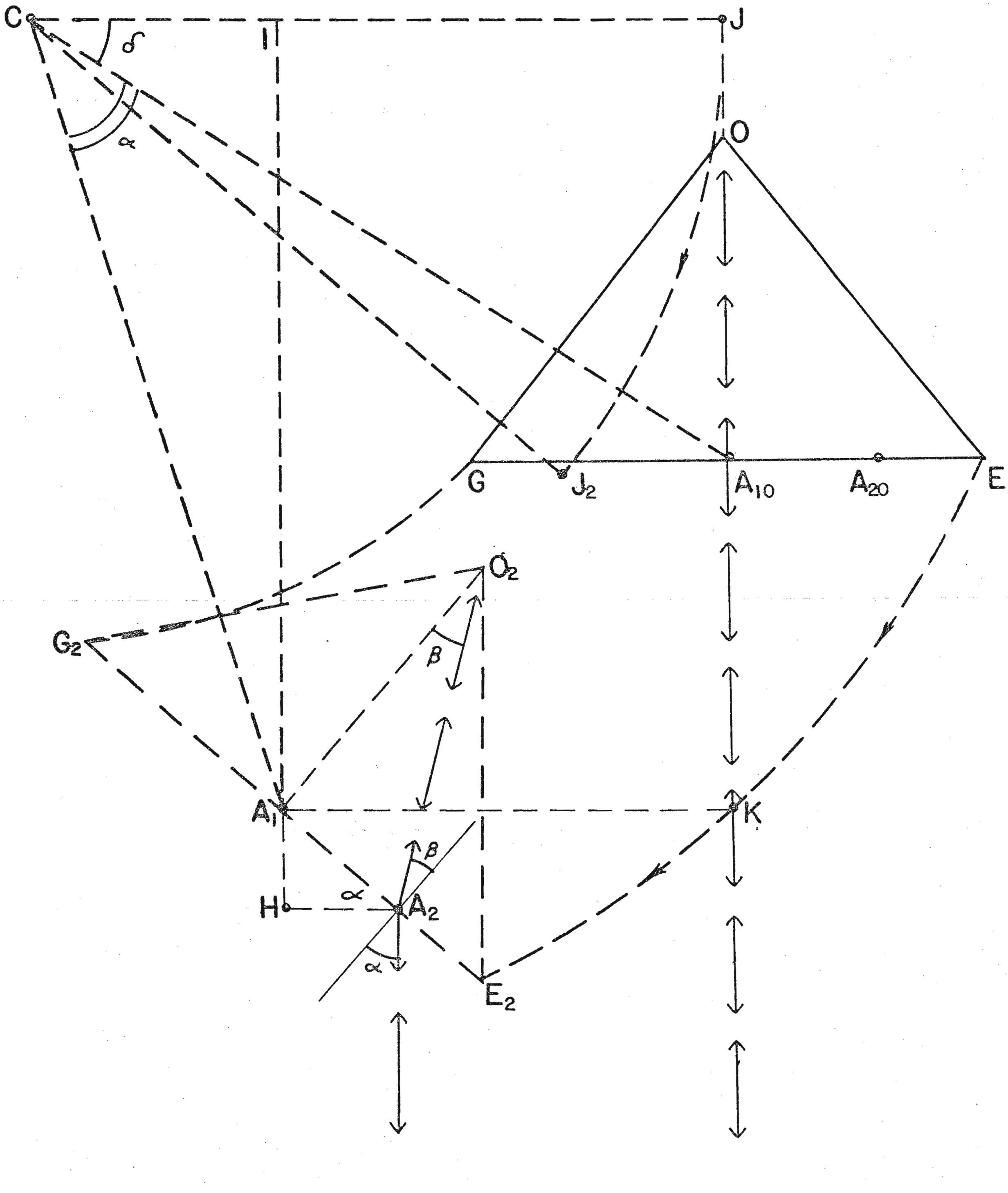


Figure 7. Outer reflector with axis of rotation C outside and behind is rotated by an angle α, with $r = JA_{10}$ and $e_H = CJ$.

The path length difference can be computed by the formulas presented above in sections 3 and 4 when:

a) nontiltable reflectors are used on slope distances,

b) reflectors are misaligned in the vertical direction,

c) reflectors are misaligned in the azimuth.

5. DETERMINATION OF THE INDEX OF REFRACTION OF CORNER REFLECTORS

The index of refraction of a corner reflector made of solid glass or plastic is not generally known except that its value is approximately 1.5. The value of the index of refraction can be determined in a well equipped optical laboratory, but it can also be determined quickly and at little cost by the following method using standard surveying instruments.

The described method requires two theodolites of which one should preferably have an autocollimating eyepiece, although it is not mandatory. The autocollimating eyepiece will allow more precise pointings to the apex of the corner reflector than direct pointing to the apex. The direct pointing of the cross hairs without autocollimation to the apex of the corner reflector is not always as precise as desired due to the irregularly rounded edges and the rounded apex of the corner reflector.

Referring to Fig. 8, center the two theodolites over points A and B which are a precisely measured distance e apart.

On top of theodolite at point B, attach V-groove (an angle iron) into which the reflector can be placed so that it is pointing in the horizontal direction to the theodolite at point A, see Fig. 10.

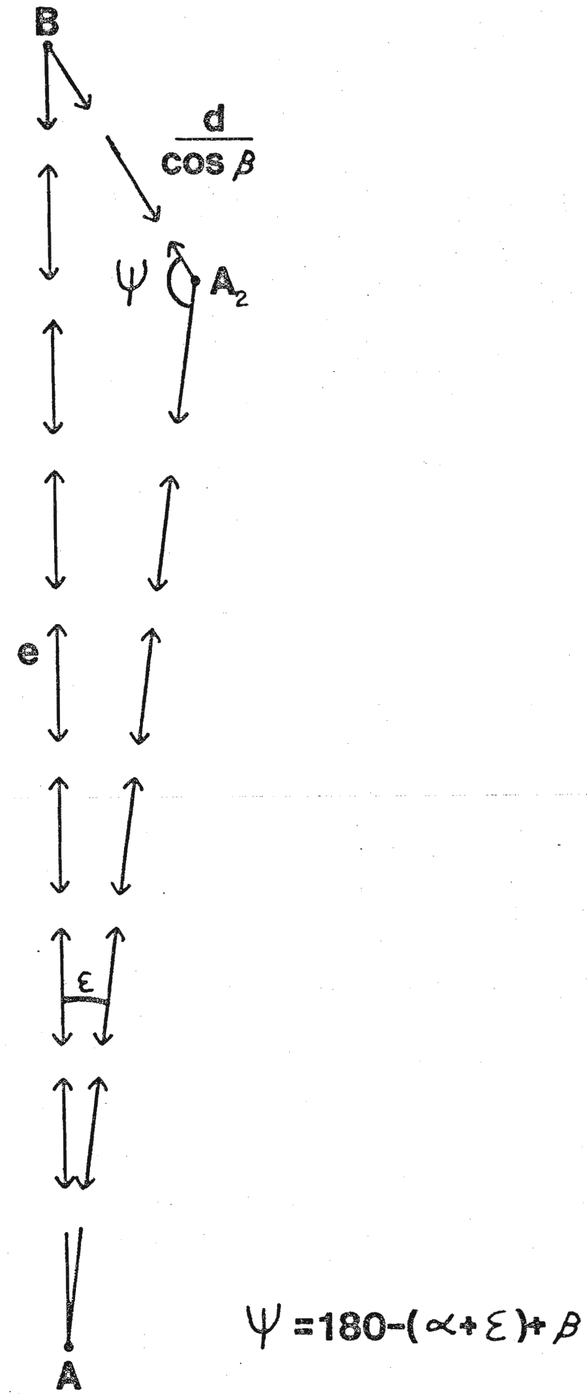


Figure 8. Theodolite with a V-groove at point B is used to rotate the reflector by a measurable angle. Theodolite at point A is used to measure angle ε. AB is the line of sight of theodolite A to the apex of the reflector before rotation, and AA_2 is the line of sight after the reflector at B has been rotated by angle α.

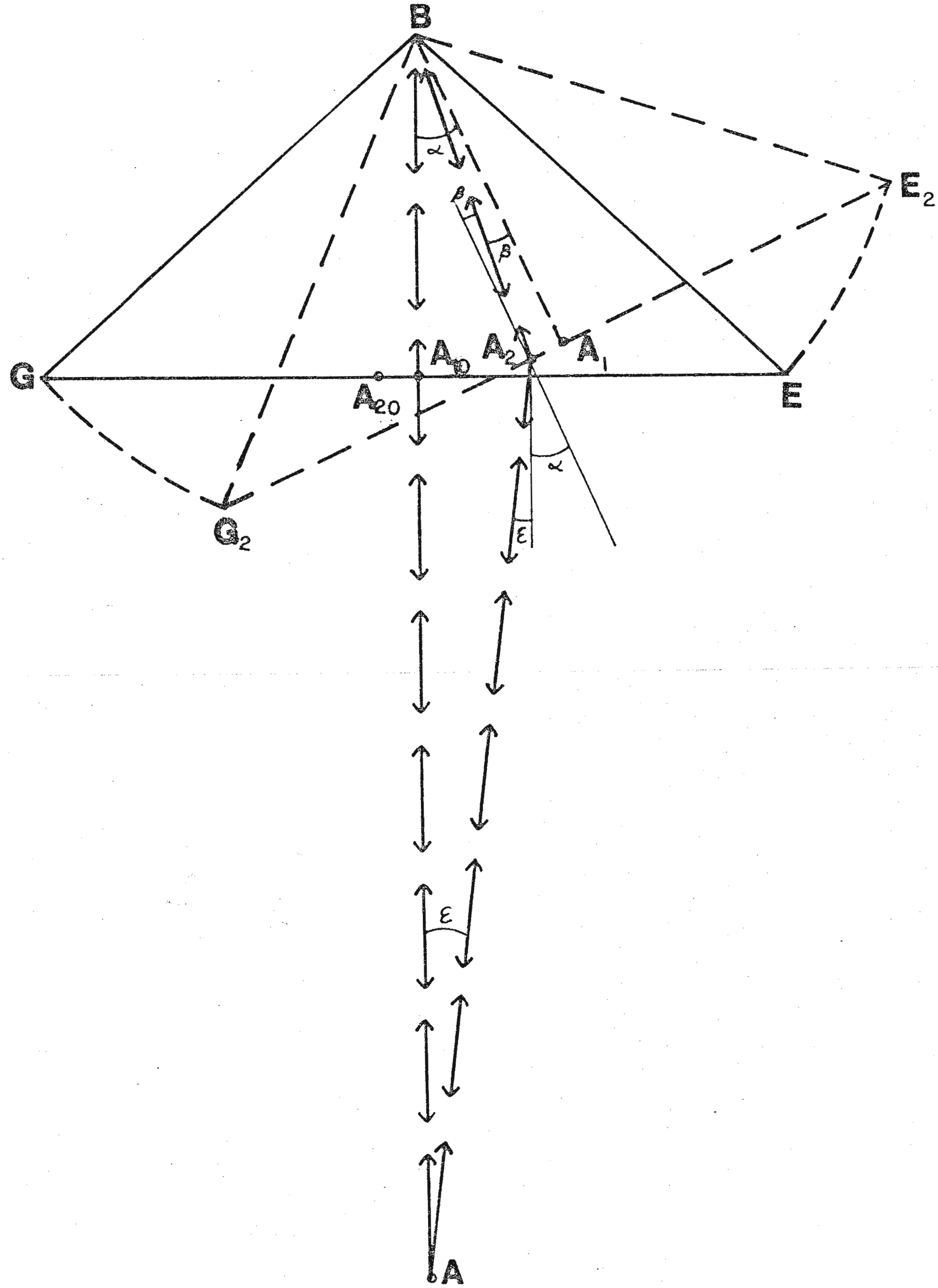


Figure 9. Referring to Fig. 8, line of sight of theodolite at point A comes to the apex at B along line $AA_{10}B$ before rotation, and along AA_2 after the reflector has been rotated by angle α.

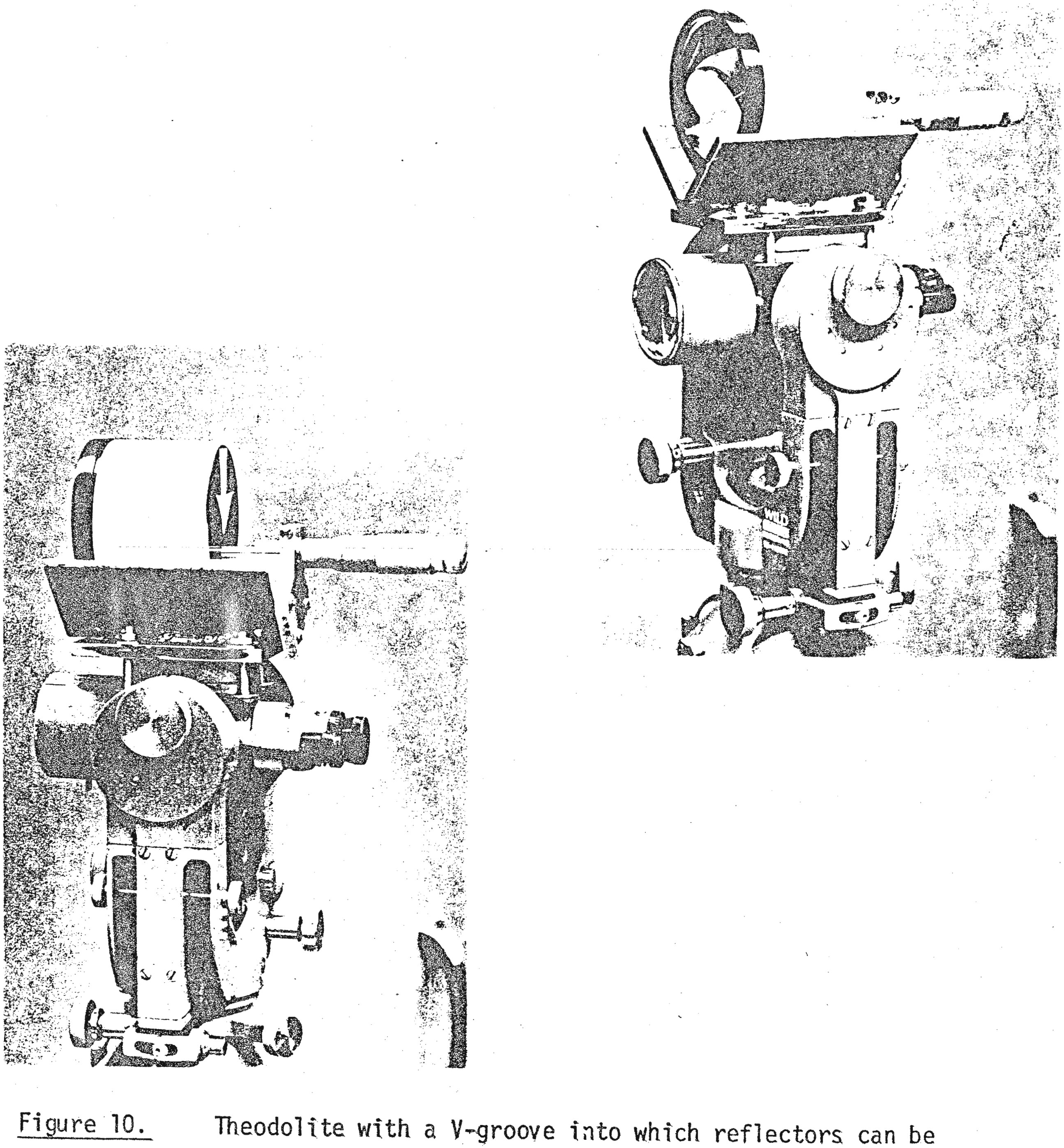


Figure 10. Theodolite with a V-groove into which reflectors can be placed, so that the apex of the reflector is on the vertical axis of the theodolite.

Adjust the tripod heights so that the center of the corner reflector is at the same height as the horizontal axis of the theodolite at point A. Level the theodolites, and check that the reflector is correctly placed.

Measure the thickness d of the corner reflector from its apex to its frontal plane, and place the apex on the vertical axis of the theodolite at point B preferably using a micrometer screw. The place where the vertical axis of theodolite B is in the V-groove can be found by turning theodolite B by $\pm 90^0$ in azimuth and observing the tip of the micrometer screw with theodolite A.

Point the theodolite B to the theodolite A, and point the cross hairs of the theodolite A to the apex of the corner reflector preferably by coinciding the autocollimated cross hairs, and record the horizontal circles of both theodolites.

Turn the theodolite B in azimuth by a measured angle α, such as 20^0, 25^0, 30^0, measuring this angle by the horizontal circle of the theodolite B, see Figs. 8 and 9.

By using the horizontal fine motion screw of the theodolite A, point the cross hairs again on the apex of the corner reflector preferably by coinciding the autocollimated cross hairs. Read the horizontal circle obtaining the angle ε in Fig. 8. Also turn the theodolite in the opposite direction by the same angle α, and repeat the measurement at A.

The detailed situation at point B can be seen from Fig. 9. When the frontal plane of the corner reflector is perpendicular to the line AB, the line of sight of the theodolite A enters the corner reflector at point A_{10} in Fig. 9. When the reflector is turned by an angle α, this point A_{10} moves to point A_1. A symmetrical situation is obtained when the turning is to the opposite direction by the same angle α.

After rotation by angle α, the line of sight of theodolite A enters the frontal plane of the reflector at point A_2. From point A_2 the line of sight is refracted to point B according to Snell's Law:

$$\frac{\sin(\alpha + \varepsilon)}{\sin\beta} = n \tag{28}$$

where $\alpha + \varepsilon$ is the angle of incidence at point A_2 and β is the angle of refraction from the surface normal in Fig. 9.

The relation between α, ε, β, and n can be obtained from ΔAA_2B seen in Figs. 8 and 9.

From Fig. 9, one can see that:

$$BA_2 = \frac{d}{\cos\beta} \tag{29}$$

where $d = BA_{10} = BA_1$, and $\measuredangle A_2BA_1 = \beta$, and from Fig. 8; $\measuredangle ABA_2 = \alpha - \beta$, and $\measuredangle AA_2B = 180^0 - (\alpha + \varepsilon) + \beta$.

Applying the Law of Sines to triangle AA_2B, one obtains

$$\frac{BA_2}{\sin\varepsilon} = \frac{e}{\sin\{180° - [(\alpha - \beta) + \varepsilon]\}} \tag{30}$$

and by using Eq. (29):

$$\frac{d}{\cos\beta\ \sin\varepsilon} = \frac{e}{\sin(\alpha - \beta + \varepsilon)} \tag{31}$$

or,

$$\frac{\sin[(\alpha + \varepsilon) - \beta]}{\cos\beta} = \frac{e \sin\varepsilon}{d} \tag{32}$$

or,

$$\frac{\sin(\alpha + \varepsilon)\cos\beta - \cos(\alpha + \varepsilon)\sin\beta}{\cos\beta} = \frac{e \sin\varepsilon}{d} \tag{33}$$

or,

$$\sin(\alpha + \varepsilon) - \cos(\alpha + \varepsilon)\tan\beta = \frac{e \sin\varepsilon}{d} \tag{34}$$

or,

$$\tan\beta = \tan(\alpha + \varepsilon) - \frac{e \sin \varepsilon}{d \cos(\alpha + \varepsilon)} \cdot \tag{35}$$

Using the measured values of α, ε, e, and d in Eq. (35), compute β, and use this in Snell's Law, Eq. (28) obtaining:

$$n = \frac{\sin(\alpha + \varepsilon)}{\sin\beta} \cdot \tag{36}$$

Rotate the reflector in the opposite direction by using the horizontal circle of theodolite B by the same angle α, measure ε, compute n in a similar manner, and then the mean value for the two n's.

The procedure, and the sensitivity of the index of refraction n as a function of d, e, α, and ε will be demonstrated by the following typical example.

Example

$d = 40.35$ mm

$e = 2300$ mm

$\alpha_R = 30^o$

$\alpha_L = 30^o$

$\varepsilon_R = 00^o12'05.''9$

$\varepsilon_L = 00^o12'07.''1$

giving: $n = \frac{n_R + n_L}{2} = \frac{1.5230 + 1.5245}{2} = 1.524$

The measured quantities were determined with the following precision (stardard errors):

$d(d) = \pm 0.2$ mm

$d(e) = \pm 1$ mm

$d(\alpha) = \pm 5''$

$d(\varepsilon) = \pm 3''$

and their effects on the index of refraction are:

$d(d) = \pm 0.2$ mm: $dn_d = \pm 0.0004$

$d(e) = \pm 1$ mm : $dn_e = \pm 0.0004$

$d(\alpha) = \pm 5''$: $dn_\alpha = \pm 0.00005$

$d(\varepsilon) = \pm 3''$: $dn_\varepsilon = \pm 0.004$

and from these

$$dn = \sqrt{(dn_d)^2 + (dn_e)^2 + (dn_\alpha)^2 + (dn_\varepsilon)^2} = \pm 0.004$$

The index of refraction in this example is then:

$$n = 1.524 \pm 0.004$$

and the value n = 1.52 will be used for this corner reflector.

When the values of n, d, r, e_H, α, and the reflector mounts are known, virtually any reflector can be used with almost any EDM instrument by the described methods.

TABLES

The following section contains tables summarizing calculated and observed path length differences for reflectors studied in this research. Dimensions and constants for reflectors and reflector mounts such as d, n and r are given in each case. The path length difference for each possible movement of the reflector was calculated using the equations presented in the previous sections. Numerous path length differences were observed with the Kern ME-3000 Mekometer and these results are also tabulated beside the computed values for easy comparison. A photograph of each reflector system is provided for reference.

The dimensions associated with the reflectors can best be seen in Figs. 11 and 12 on pages 33 and 34. In cases where the quantities r_V and r_H are the same, an r with no subscript is used.

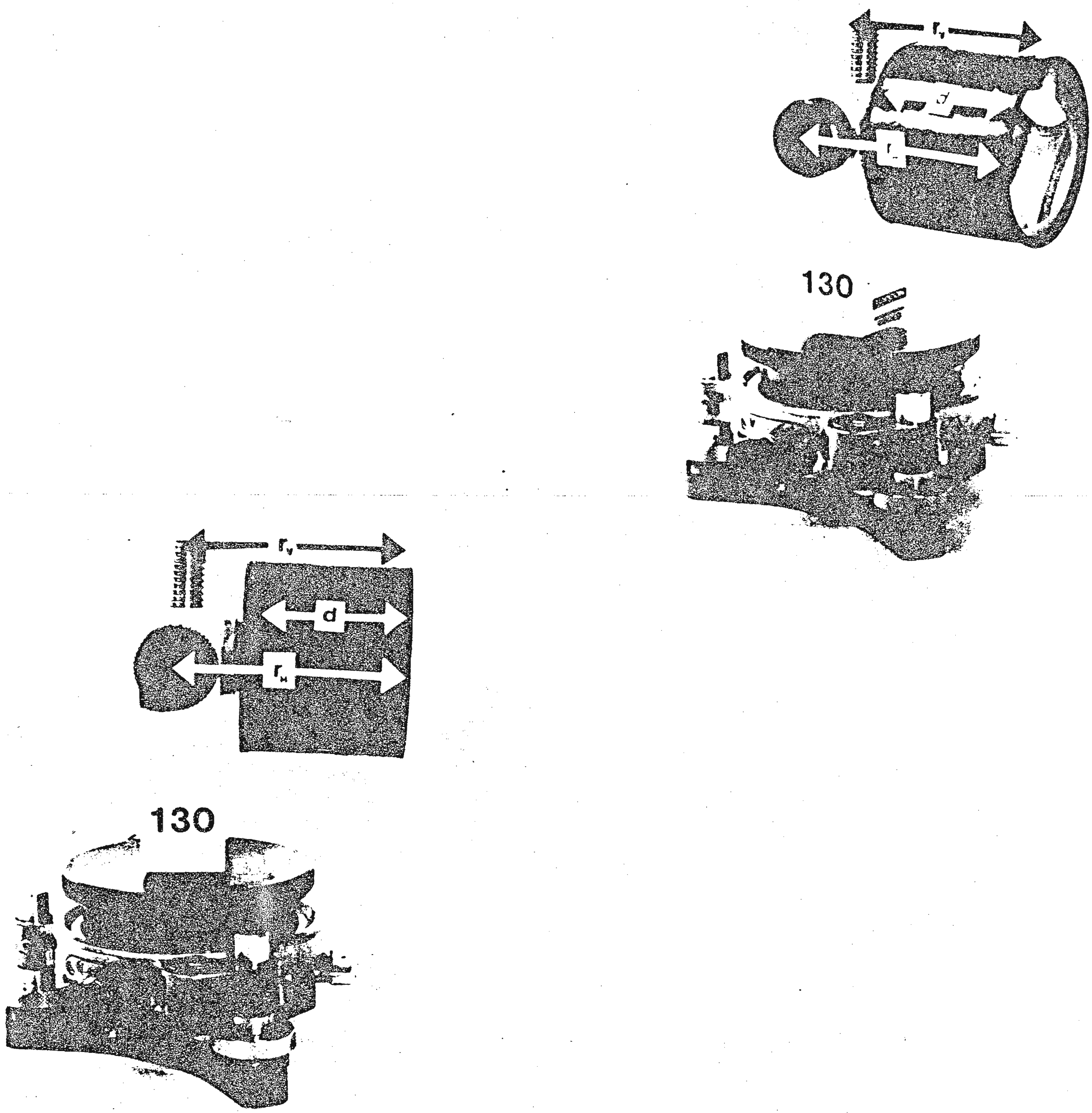


Figure 11. Dimensions associated with a single reflector.

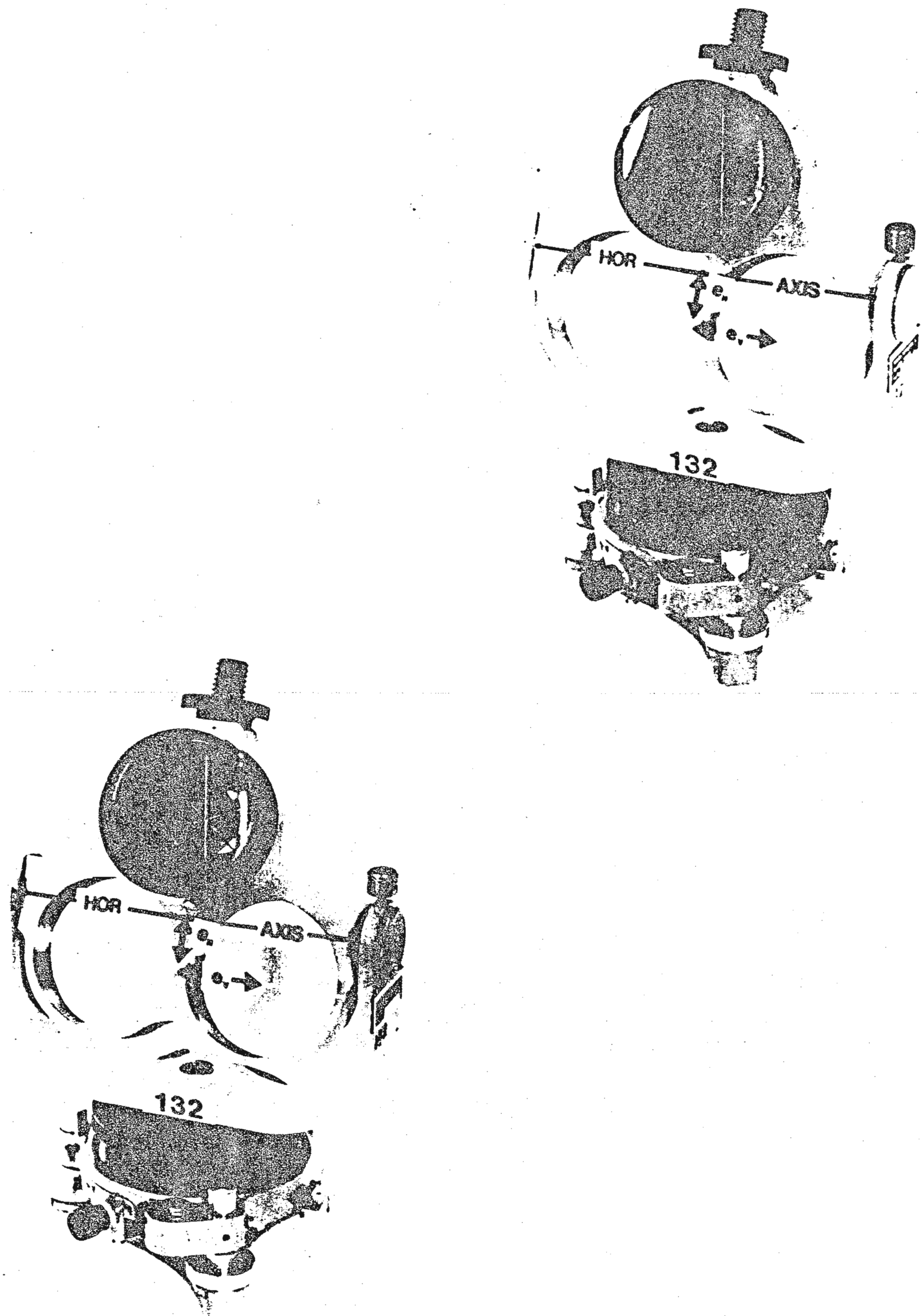


Figure 12. Dimensions associated with multiple reflectors.

Reflector #111

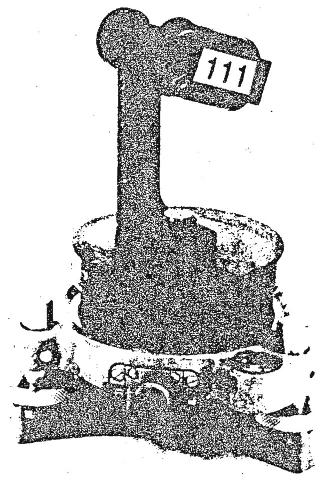


Table 1 Path Length Difference in Azimuth and Vertical

n = 1.54 (assumed) α = angle of incidence
r = -1.95* mm. PLD = path length difference
d = 18.60 mm.

α	Computed PLD in Azimuth and Vertical	Observed PLD in Azimuth	Observed PLD in Vertical
	mm	mm	mm
1°	-0.002		
2°	-0.009		
3°	-0.019		
4°	-0.034		
5°	-0.053		
10°	-0.212	0.0	0.2
15°	-0.474	-0.5	
20°	-0.833		-0.7
25°	-1.282		
30°	-1.813		
35°	-2.414		
40°	-3.071		

* negative denotes axis in front of reflector

Reflector #112

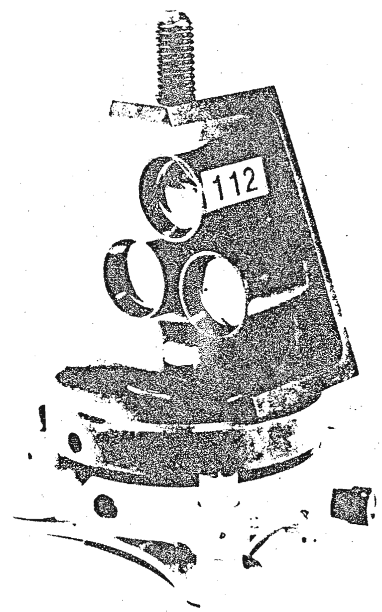


Table 2a Path Length Difference in Azimuth and Vertical* for the Top Reflector

n = 1.54 (assumed)
r = -1.80 mm
d = 18.60 mm

e_H = 18.65 cm
α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth and Vertical	Observed PLD in Azimuth
	mm	mm
1°	-0.002	
2°	-0.008	
3°	-0.019	
4°	-0.034	
5°	-0.053	
10°	-0.210	-0.2
15°	-0.469	
20°	-0.824	-0.9
25°	-1.268	
30°	-1.793	
35°	-2.386	
40°	-3.036	

* The reflectors do not tilt so vertical PLD was not observed

Table 2b   Path Length Difference in Azimuth and Vertical* for a Lower Reflector

α	Computed PLD in Azimuth	Observed PLD in Azimuth
	mm	mm
1°	-0.328	
2°	-0.659	
3°	-0.995	
4°	-1.335	
5°	-1.678	
10°	-3.449	-3.3
15°	-5.296	
20°	-7.203	-6.5
25°	-9.150	
30°	-11.118	
35°	-13.084	
40°	-15.024	

*The reflectors do not tilt so vertical PLD was not observed

Reflector #121

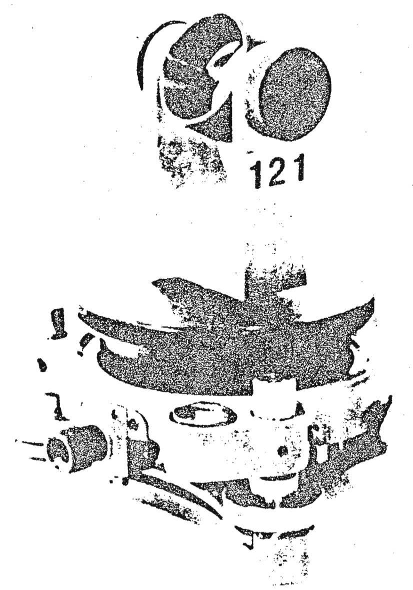


Table 3 Path Difference in Azimuth and Vertical

n = 1.59
r = -1.40 mm
d = 19.10 mm

α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth and Vertical*
	mm
1°	-0.002
2°	-0.008
3°	-0.018
4°	-0.033
5°	-0.051
10°	-0.203
15°	-0.453
20°	-0.795
25°	-1.224
30°	-1.728
35°	-2.298
40°	-2.920

* unable to observe PLD with the Mekometer 3000

Reflector #122

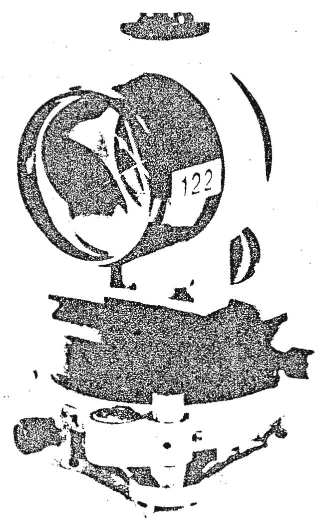


Table 4 Path Length Difference in Azimuth and Vertical

$n = 1.56$ α = angle of incidence
$r = 46.55$ mm PLD = path length difference
$d = 50.38$ mm

α	Computed PLD in Azimuth and Vertical*
	mm
1°	0.002
2°	0.009
3°	0.020
4°	0.035
5°	0.054
10°	0.219
15°	0.497
20°	0.895
25°	1.422
30°	2.090
35°	2.913
40°	3.909

* unable to observe PLD with the Mekometer 3000

Reflector # 123

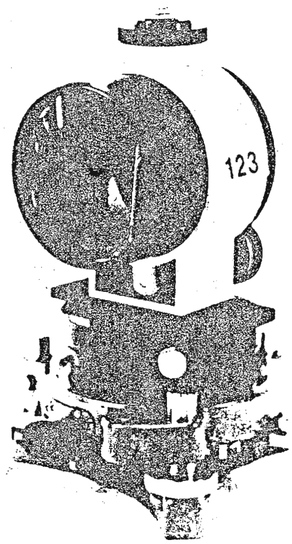


Table 5 Path Length Difference in Azimuth and Vertical

n = 1.54 α = angle of incidence
r = 47.70 mm   PLD = path length difference
d = 52.10 mm

α	Computed PLD in Azimuth and Vertical*
	mm
1°	0.002
2°	0.008
3°	0.019
4°	0.034
5°	0.053
10°	0.213
15°	0.484
20°	0.873
25°	1.389
30°	2.044
35°	2.854
40°	3.836

* unable to observe PLD with the Mekometer 3000

Reflector # 124

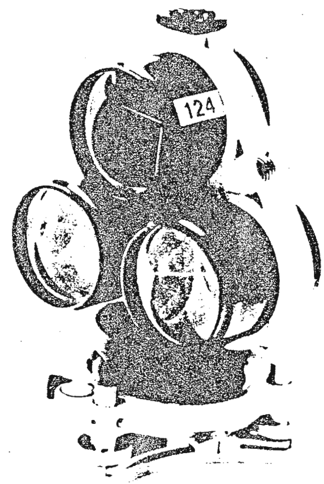


Table 6a Path Length Difference in Azimuth and Vertical* for the Top Reflector

n = 1.56 (assumed)
r = 47.15 mm
d = 49.90 mm

e_V = 40.85 mm
α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth and Vertical
	mm
1°	0.002
2°	0.009
3°	0.021
4°	0.037
5°	0.058
10°	0.233
15°	0.528
20°	0.950
25°	1.507
30°	2.210
35°	3.074
40°	4.116

* The reflectors do not tilt but azimuthal PLD is equal to the vertical PLD

Table 6b Path Length Difference in Azimuth for a Lower Reflector

α	Computed PLD in Azimuth
	mm
1°	-0.711
2°	-1.416
3°	-2.117
4°	-2.813
5°	-3.502
10°	-6.861
15°	-10.045
20°	-13.022
25°	-15.757
30°	-18.215
35°	-20.356
40°	-22.142

Reflector # 125

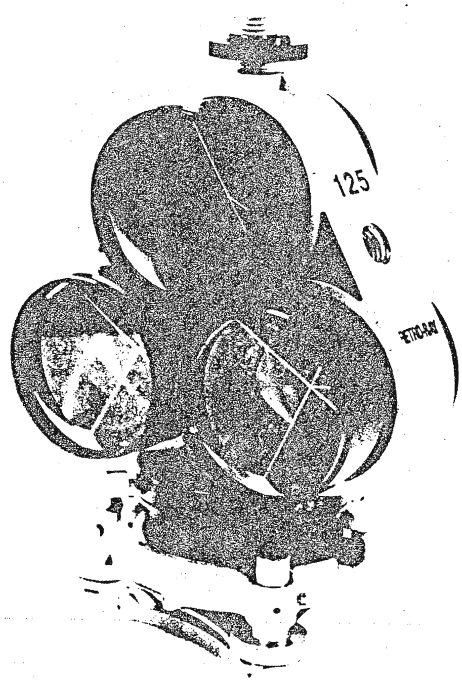


Table 7a Path Length Difference in Azimuth and Vertical* for the Top Reflector

$n = 1.54$ (assumed)
$r = 47.00$ mm
$d = 52.10$ mm

$e_v = 40.65$ mm
α = angle of incidence
PLD = path length difference

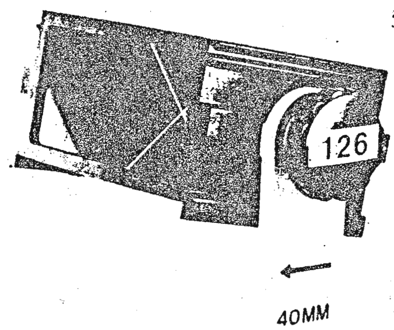
α	Computed PLD in Azimuth and Vertical
	mm
1°	0.002
2°	0.008
3°	0.018
4°	0.032
5°	0.050
10°	0.202
15°	0.460
20°	0.831
25°	1.323
30°	1.950
35°	2.727
40°	3.673

* The reflectors do not tilt but azimuthal PLD is equal to the vertical PLD

Table 7b Path Length Difference in Azimuth for a Lower Reflector

α	Computed PLD in Azimuth
	mm
1°	-0.708
2°	-1.411
3°	-2.109
4°	-2.803
5°	-3.493
10°	-6.856
15°	-10.061
20°	-13.072
25°	-15.856
30°	-18.375
35°	-20.589
40°	-22.457

Reflector # 126



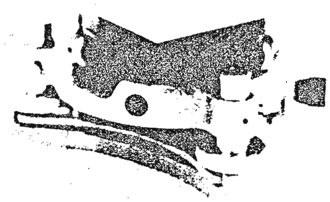
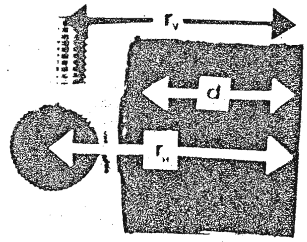


Table 8 Path Length Difference in Azimuth and Vertical

$n = 1.53$ $d = 76.00$ mm
$r_V = 74.00$ mm α = angle of incidence
$r_H = 41.90$ mm PLD = path length difference

α	Computed PLD in Azimuth	Computed PLD in Vertical
	mm	mm
1°	0.004	-0.001
2°	0.015	-0.005
3°	0.033	-0.011
4°	0.059	-0.019
5°	0.093	-0.029
10°	0.373	-0.115
15°	0.846	-0.248
20°	1.520	-0.416
25°	2.409	-0.598
30°	3.530	-0.771
35°	4.903	-0.903
40°	6.553	-0.957

Reflector # 130



130

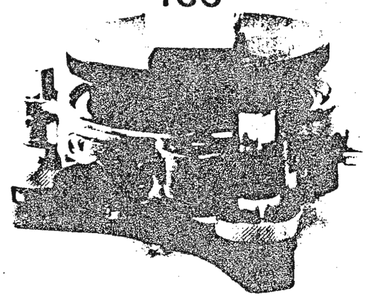


Table 9 Path Length Difference in Azimuth and Vertical

$n = 1.54$
$r = 80.15$ mm
$d = 52.15$ mm

α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth and Vertical	Observed PLD in Azimuth	Observed PLD in Vertical
	mm	mm	mm
1°	0.007		
2°	0.028		
3°	0.064		
4°	0.114		
5°	0.177		
10°	0.710	0.6	0.5
15°	1.599		
20°	2.845	2.8	3.0
25°	4.453		
30°	6.425	6.8	6.6
35°	8.767	9.1	
40°	11.464		11.7

Reflector # 131

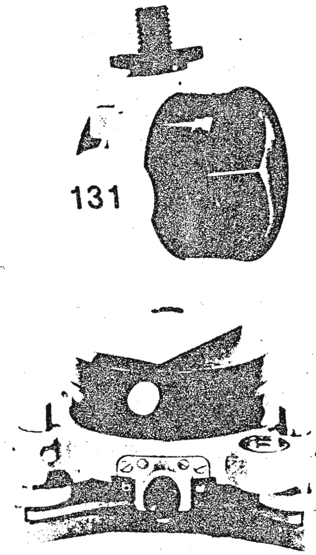


Table 10 Path Length Difference in Azimuth and Vertical

n = 1.51
r = 35.70 mm
d = 51.25 mm

α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth and Vertical	Observed PLD in Azimuth	Observed PLD in Vertical
	mm	mm	mm
1°	0.000		
2°	0.001		
3°	0.002		
4°	0.004		
5°	0.007		
10°	0.029	0.0	0.1
15°	0.071		
20°	0.142	0.3	0.1
25°	0.252		
30°	0.417	0.8	0.4
35°	0.656	1.1	
40°	0.990		0.6

Reflector # 132

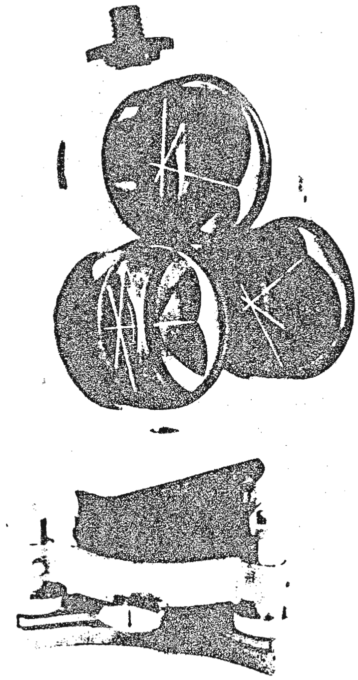


Table 11a Path Length Difference in Azimuth and Vertical for the Top Reflector

$n = 1.50$
$r = 36.75$ mm
$d = 51.25$ mm
$e_V = 37.00$ mm

$e_H = 22.40$ mm (lower reflectors)
$e_H = 42.80$ mm (top reflector)
α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth	Observed PLD in Azimuth	Computed PLD in Vertical	Observed PLD in Vertical
	mm	mm	mm	mm
1°	0.000		-0.747	
2°	0.002		-1.492	
3°	0.004		-2.236	
4°	0.006		-2.979	
5°	0.010		-3.720	
10°	0.041	-0.1	-7.391	-7.0
15°	0.099		-10.978	
20°	0.191	0.1	-14.447	-14.6
25°	0.329		-17.759	
30°	0.527	1.1	-20.873	-21.0
35°	0.804		-23.745	-23.2
40°	1.182	1.4	-26.330	

Table 11b Path Length Difference in Azimuth and Vertical for the Lower Reflectors

α	Computed PLD in Azimuth	Observed PLD in Azimuth	Computed PLD in Vertical	Observed PLD in Vertical
	mm	mm	mm	mm
1°	-0.645		-0.391	
2°	-1.290		-0.780	
3°	-1.933		-1.169	
4°	-2.575		-1.556	
5°	-3.215		-1.942	
10°	-6.384	-6.3	-3.848	-3.4
15°	-9.477		-5.698	
20°	-12.463	-12.6	-7.470	-6.5
25°	-15.308		-9.138	
30°	-17.973	-18.2	-10.673	-9.2
35°	-20.418		-12.044	-10.2
40°	-22.601	-23.1	-13.217	

Reflector # 137

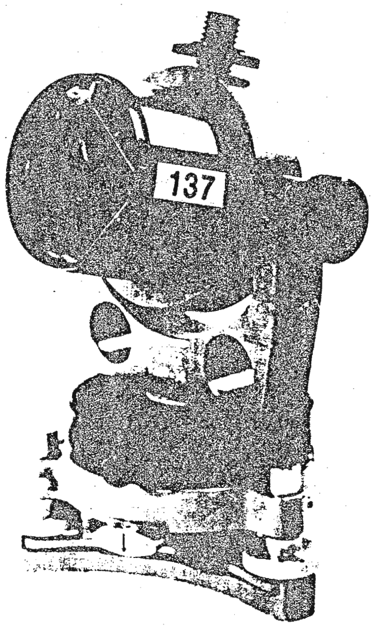


Table 12 Path Length Difference in Azimuth and Vertical

$n = 1.53$ α = angle of incidence
$r = 53.35$ mm PLD = path length difference
$d = 56.85$ mm

α	Computed PLD in Azimuth and Vertical
	mm
1°	0.002
2°	0.010
3°	0.022
4°	0.040
5°	0.062
10°	0.248
15°	0.564
20°	1.016
25°	1.614
30°	2.372
35°	3.305
40°	4.433

Holder #133H with Reflector #138

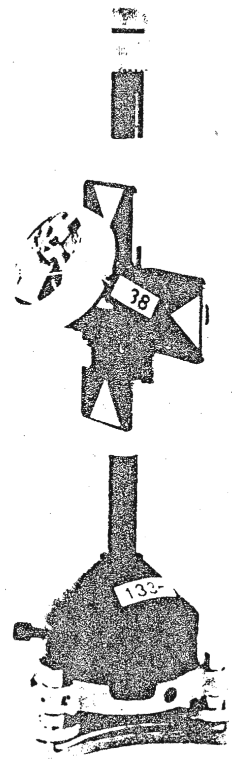


Table 13 Path Length Difference in Azimuth and Vertical

n = 1.54 (assumed)
r = 69.05
d = 41.05 mm

α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth and Vertical	Observed PLD in Azimuth	Observed PLD in Vertical
	mm	mm	mm
1°	0.006		
2°	0.026		
3°	0.058		
4°	0.103		
5°	0.161		
10°	0.646	0.3	
15°	1.454	1.1	1.4
20°	2.585	2.2	
25°	4.042		
30°	5.826		5.6
35°	7.939	7.3	
40°	10.385		

Holder # 132H with Reflector # 135

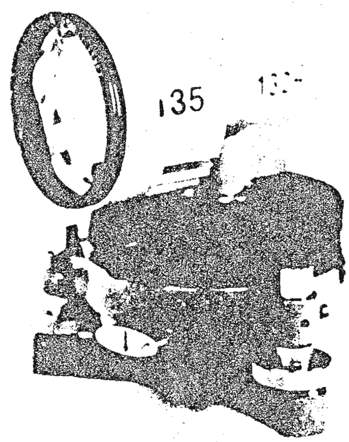


Table 14 Path Length Difference in Azimuth and Vertical*

n = 1.51
r = 62.45 mm
d = 40.89 mm

α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth and Vertical	Observed PLD in Azimuth
	mm	mm
1°	0.005	
2°	0.022	
3°	0.048	
4°	0.086	
5°	0.135	
10°	0.539	0.7
15°	1.214	
20°	2.161	2.2
25°	3.383	
30°	4.884	5.1
35°	6.666	
40°	8.737	

* The holder does not tilt but azimuthal PLD is equal to vertical PLD.

Holder # 133H with Reflector # 135

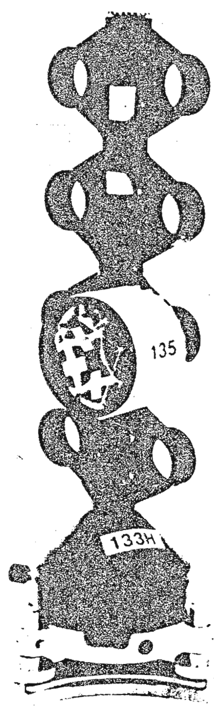


Table 15 Path Length Difference in Azimuth and Vertical*

$n = 1.51$
$r = 62.45$ mm
$d = 40.89$ mm

$e_V = 35.60$ mm
α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth and Vertical	Observed PLD in Azimuth
	mm	mm
1°	-0.616	
2°	-1.221	
3°	-1.815	
4°	-2.397	
5°	-2.968	
10°	-5.643	-5.8
15°	-8.000	
20°	-10.014	-10.6
25°	-11.662	
30°	-12.916	-13.2
35°	-13.753	-13.9
40°	-14.146	

* The holder does not tilt but azimuthal PLD is equal to vertical PLD.

Holder #134H with Reflector #135

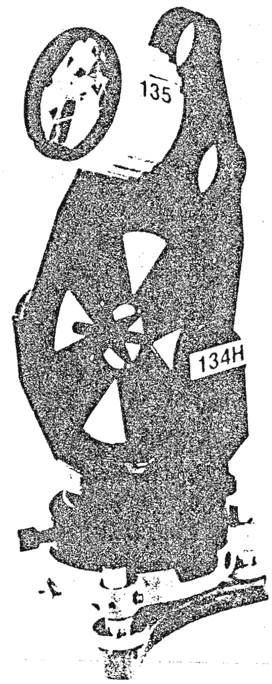


Table 16a Path Length Difference in Azimuth for the Top Two Mountings

$n = 1.51$
$r = 62.50$ mm
$d = 40.89$ mm

$e_V = 38.00$ mm
α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth	Observed PLD in Azimuth
	mm	mm
1°	0.005	
2°	0.022	
3°	0.049	
4°	0.086	
5°	0.135	
10°	0.540	0.6
15°	1.216	
20°	2.165	2.3
25°	3.388	
30°	4.890	5.8
35°	6.675	
40°	8.749	

Table 16b Path Length Difference in Azimuth for the Lower Two Mountings

α	Computed PLD in Azimuth	Observed PLD in Azimuth
	mm	mm
1°	-0.658	
2°	-1.305	
3°	-1.940	
4°	-2.564	
5°	-3.177	
10°	-6.059	-6.1
15°	-8.619	
20°	-10.832	-11.8
25°	-12.671	
30°	-14.110	-17.0
35°	-15.121	
40°	-15.677	

Mekometer Single Reflector

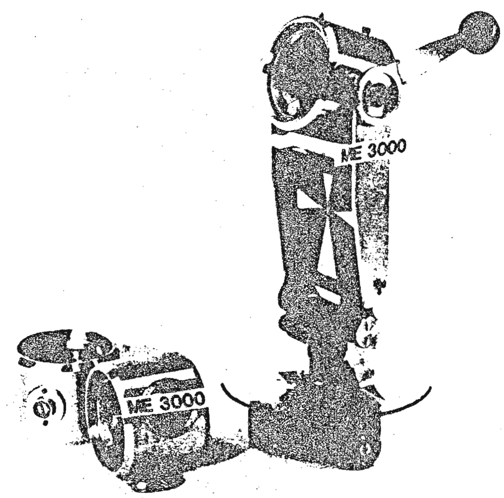


Table 17 Path Length Difference in Azimuth and Vertical

n = 1.52 α = angle of incidence
r = 16.90 mm PLD = path length difference
d = 40.35 mm

α	Computed PLD in Azimuth and Vertical	Observed PLD in Azimuth	Observed PLD in Vertical
	mm	mm	mm
1°	-0.001		
2°	-0.005		
3°	-0.011		
4°	-0.020		
5°	-0.031		
10°	-0.124	-0.1	-0.1
15°	-0.272		
20°	-0.469	-0.3	-0.4
25°	-0.704		
30°	-0.961	-0.6	-0.9
35°	-1.225		
40°	-1.473	-0.8	-1.5

Mekometer Triple Reflector

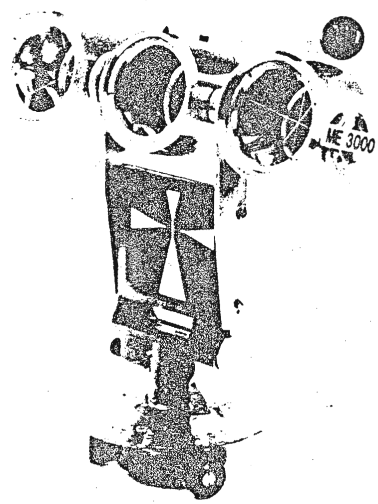


Table 18 Path Length Difference in Azimuth and Vertical*

n = 1.52
r = 17.30 mm
d = 40.35 mm

e_V = 98.75 mm

α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth	Observed PLD in Azimuth
	mm	mm
1°	-1.725	
2°	-3.452	
3°	-5.181	
4°	-6.911	
5°	-8.642	
10°	-17.286	-16.3
15°	-25.865	
20°	-34.304	-31.9
25°	-42.531	
30°	-50.470	-46.6
35°	-58.046	
40°	-65.182	-61.0

* PLD in vertical is equal to the PLD in vertical for a single prism.

Wild DI-10 Reflector

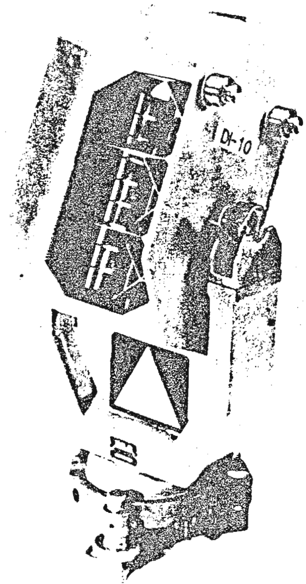


Table 19 Path Length Difference in Azimuth and Vertical for the Center Reflector

n = 1.54 (assumed)
r = 20.50 mm
d = 60.10 mm

α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth and Vertical
	mm
1°	-0.003
2°	-0.011
3°	-0.025
4°	-0.045
5°	-0.070
10°	-0.279
15°	-0.618
20°	-1.075
25°	-1.633
30°	-2.268
35°	-2.952
40°	-3.652

HP Triple Reflector

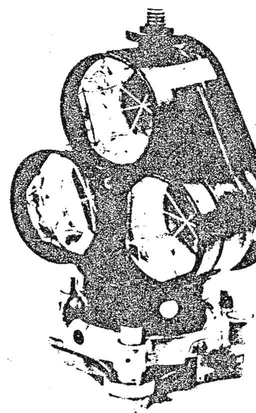


Table 20a Path Length Difference in Azimuth for the Top Reflector

n = 1.55
r = 34.65 mm
d = 41.35 mm

e_V = 37.85 mm
α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth	Observed PLD in Azimuth
	mm	mm
1°	0.001	
2°	0.005	
3°	0.011	
4°	0.019	
5°	0.030	
10°	0.123	-0.2
15°	0.281	
20°	0.510	0.4
25°	0.818	
30°	1.216	1.6
35°	1.717	
40°	2.336	

Table 20b Path Length Difference in Azimuth for the Lower Reflector

α	Computed PLD in Azimuth	Observed PLD in Azimuth
	mm	mm
1°	-0.659	
2°	-1.316	
3°	-1.970	
4°	-2.621	
5°	-3.268	
10°	-6.450	-6.4
15°	-9.515	
20°	-12.436	-12.7
25°	-15.178	
30°	-17.709	-18.4
35°	-19.993	-21.2
40°	-21.994	

HP Single Reflector

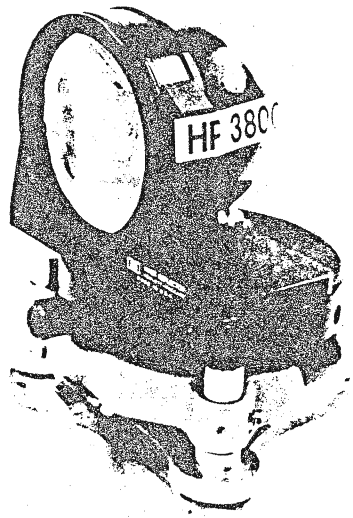


Table 21 Path Length Difference in Azimuth and Vertical*

n = 1.54 (assumed)
r = 47.55 mm
d = 50.70 mm

α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth and Vertical
	mm
1°	0.002
2°	0.009
3°	0.020
4°	0.036
5°	0.056
10°	0.224
15°	0.510
20°	0.918
25°	1.457
30°	2.141
35°	2.982
40°	3.998

* The reflector does not tilt but azimuthal PLD is equal to the vertical PLD

DI-3S Single Reflector

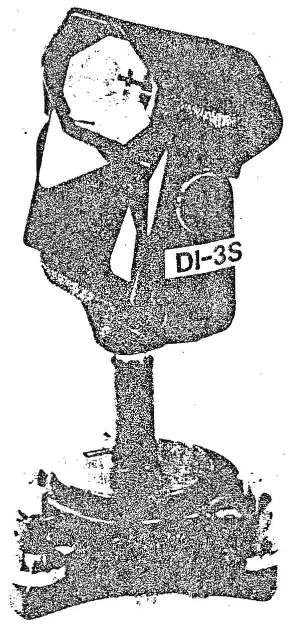


Table 22 Path Length Difference in Azimuth and Vertical

n = 1.54 (assumed)
r = 23.15 mm
d = 58.69 mm

e_H = 36.55 mm
α = angle of incidence
PLD = path length difference

α	Computed PLD in Azimuth	Computed PLD in Vertical	Observed PLD in Vertical
	mm	mm	mm
1°	-0.002	-0.640	
2°	-0.009	-1.285	
3°	-0.020	-1.933	
4°	-0.036	-2.586	
5°	-0.057	-3.242	
10°	-0.225	-6.572	-6.4
15°	-0.497	-9.957	
20°	-0.861	-13.362	-12.6
25°	-1.301	-16.748	-16.3
30°	-1.795	-20.070	
35°	-2.316	-23.280	
40°	-2.834	-26.327	

CONCLUSION

By using the formulas derived in this report, path length differences have been computed for many single and multiple reflectors. These theoretical values have been checked through actual measurements with the Kern ME-3000 Mekometer. The agreement was excellent. It is believed that the largest encountered differences between the computed and measured path length differences (2 to 4 mm) were mostly due to inaccuracies in measuring the angles of misalignment of the reflector. Most of the computed and measured path length differences agree within one millimeter.

It can be seen from the previous tables that different reflectors produce different path length differences for a given angle of misalignment. It also should be noted that a given misalignment in azimuth and in the vertical may produce path length differences which may differ by a substantial amount.

For example, a 30^{0} misalignment of Mekometer ME-3000 triple reflector in vertical will produce a path length difference of one millimeter but the same misalignment in azimuth will cause path length differences for the three reflectors to be $\pm$ 50 mm, - 1 mm, and $\mp$ 50 mm, respectively. This can be seen on pages 56 and 57.

For a 30^{0} misalignment, the other tabulated reflectors produce path length differences ranging from one to $\pm$ 18 millimeters.

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REFERENCES

1. Kivioja, L. A., "The EDM Corner Reflector Constant Is Not Constant", Surveying and Mapping, pp. 143-155, June 1978, and pp. 161-162, June 1979.

2. Konttinen, R., "Determination of Zero and Scale Corrections of the Mekometer on Nummela Standard Baseline", Proc. IAG Symposium High Precision Geodetic Length Measurements, pp. 255-279, June 1978, Helsinki.

3. Meier-Hirmer, B., "Präzisionsstreckenmessung mit dem Mekometer ME 3000", Allgemeine Vermessungs-Nahrichten, Nr. 2/75, Karlsruhe. (Translated to English by Kern, Inc.)

4. Rüeger, J. M., "Remarks on the Joint Determination of Zero Error and Cyclic Error for EDM Instrument Calibration", The Australian Surveyor, pp. 96-103, June 1976.

5. Rüeger, J. M., "Misalignment of EDM Reflectors and Its Effects", The Australian Surveyor, March 1978.

6. Rüeger, J. M. and Brunner, F. K. "Practical Results of EDM Height Traversing", The Australian Surveyor, pp. 363-372, June 1981.