AZIMUTHS IN CONTROL SURVEYS

ABSTRACT

With the advent of GPS for control surveying, a problem has arisen. Many existing control stations established by conventional methods (i.e. first order triangulation stations) are in locations not suitable for direct occupation by GPS equipment. Eccentric observations are often possible, with the eccentric station located up to 100 m away. In order to connect the eccentric station to the main station, an azimuth is necessary. Occasionally, a companion GPS station is set, and the angle is measured. More often, it is necessary to observe an astronomic azimuth. Various methods are explored for determining azimuth, with a discussion of the theory of each method, equipment and procedures, and accuracy attainable. The different methods examined include astronomic, gyrotheodolite, azimuth pairs by GPS, and use of azimuth marks. Existing Standards and Specifications address the issue of azimuths only in the context of controlling traverses and triangulation networks, with accuracies of 0.5" to 2" the goal. This paper addresses the issue of lower accuracy azimuths, in the range of 5" to 20". Also discussed is the difference between astronomic, geodetic, and grid north. A discussion is made for potential inclusion in the "Input Formats and Specifications of The National Geodetic Survey Data Base" of the different types of azimuths besides the presently accepted polaris azimuth, when the distances are short and the required accuracy is lower.

INTRODUCTION

Over the years, various methodologies have been used to obtain azimuth control when extending horizontal control whether by traverse or triangulation. In the eastern part of the US, old photographs show that in the early part of this century, much of the area had few trees, having been timbered and cleared for farming. Many of the triangulation stations, located on high hilltops, had good visibility to an adjacent station or at least to an azimuth mark. Over the years, most of the rural areas have reforested, and the urban areas have become built up. It is a rare occurrence now that a recovered 1st order station has any kind of available (i.e. visible from the ground) azimuth reference. This was probably one of the main reasons why local surveyors often did not make ties to this control. Instead, many surveys used assumed datums with assumed azimuths. Some surveyors simply backsighted the nearby reference marks, thereby introducing large errors into their surveys.

When GPS arrived in the mid 1980's, there was precious little satellite coverage in a day. When establishing a survey network to be used by conventional equipment, the clients often specified that an intervisible azimuth mark was to be established. Because of the high cost of GPS equipment and the short GPS window, usually stations were established individually, and astronomic observations were then made at some or all of the stations to establish azimuth marks. Now, with 24 hour coverage and shortened observation times, stations are frequently established in pairs. Still, it is not always possible to set an intervisible station a sufficient distance away in a location that is suitable for GPS.

John Hamilton hamilton@terrasurv.com
Geodetic Engineer 412-341-5620

Before the establishment of HARN (High Accuracy Reference Network) stations, it was necessary (and still sometimes is) to use existing triangulation stations for control, which are often located in wooded or otherwise obstructed locations. The NGS (and its predecessor, the USC&GS) utilized Bilby towers, ranging in height up to 110+ feet, to achieve intervisibility between main scheme stations. At many of these stations, although a direct occupation is not possible, it is possible to locate an eccentric GPS station less than 50 meters away. This necessitates observing an astronomic azimuth between the eccentric and the existing station. Another common situation is when a survey station, for example a property corner, is not occupiable by GPS, but it is possible to establish a GPS station nearby. The Federal Geodetic Control Committee (FGCC, now FGCS) specifications for conventional control surveys, last updated in September 1984, do not specifically address these situations. However, the document does specify that Astronomic Azimuths are required every X number of segments in a traverse, and X number of triangles in a triangulation network, depending on the order of the survey. The azimuths are implied to be polaris azimuths (or other star) by the "Observations per Night" and "Number of Nights" criteria, as well as the standard deviation (0.45" for first order up to 1.7" for third order) required. While this accuracy is certainly important for controlling extensive traverse/triangulation nets, it is much higher than necessary for a short eccentric reduction. The difficulty of working at night, and the unfamiliarity of using Polaris had the following effects:

- assumed datums, because the surveyor did not have survey orientation to start and end the survey, and therefore did not use the geodetic control
- use of nearby (inaccurate) reference marks for azimuth control
- assuming an azimuth for a traverse starting leg, then rotating the entire traverse to fit the ending coordinates, sometimes including angular blunders in the network
- use of intersection stations (which are third order, and may be subject to movement)
- having to go much further for control for GPS, or using lower order stations

All of these resulted in survey networks which were less (sometimes much less) accurate than would have otherwise been possible.

The purpose of this paper is to review the concepts for determining azimuths, particularly astronomic methods, and to give an idea of the accuracies and complexities of each method. The azimuth accuracy required at a distance of 30 m to achieve an accuracy of 0.003 m is 20", which, as will be shown, is easily obtained using any visible object in space, whether polaris, other stars, sun, moon, or planets, when proper observational and computation procedures are followed. What follows is often a simplification, appropriate to the accuracies discussed. Corrections applied to first order azimuths, such as polar motion, and diurnal aberration, are not considered.

WHAT IS AN AZIMUTH?

An azimuth is defined as a horizontal angle reckoned clockwise from the meridian. There are several types of "north". Astronomic north is with respect to the astronomic meridian, which varies from point to point in an irregular manner under the influence of gravity. Geodetic north is with respect to the ellipsoidal meridian, which differs from the astronomic meridian by a varying amount. Grid north is with respect to a central meridian of a mapping projection. Finally, magnetic north is the direction of the magnetic field of the earth. This also varies from point to point (and over time) in an irregular manner. The astronomic north is often used (incorrectly) in geodetic computations, although the geodetic north is what is required. The difference between the two (Laplace correction) is due to the deflection of the vertical in the prime vertical, caused by variations in gravity. Actually, the Laplace correction is a function of the east-west slope of the geoid. Astronomic north does have some uses, for example, to align inertial navigation systems. The magnetic declination at a point can be interpolated using a model from the USGS. However, the accuracy of determining geodetic north using a compass is about 1°, at best, and will not be addressed further. This paper will deal with the first three types of north mentioned, namely astronomic, geodetic, and grid, and the relationships between the three. For explanation purposes, P_1 is the standpoint (i.e. occupied by the theodolite) and P_2 is the forepoint (i.e. occupied by a target).

The astronomic azimuth is defined as the angle measured in the horizontal plane between the astronomic meridian of P_1 and the vertical plane spanned by the vertical at P_1 and by point P_2. This value is physically measurable, and is what we measure when we observe the sun, polaris, or other objects in space using an instrument which measures with respect to the local plumb line (i.e. theodolite). The astronomic azimuth can also be determined by using a gyro-theodolite, such as the Wild GAK-1.

The geodetic azimuth is defined as the angle measured in the horizontal plane between the ellipsoidal meridian of P_1 and the vertical plane spanned by the normal to P_1 and by point P_2. This appears to be the same as the definition given above for astronomic azimuth. There are two differences, namely the meridian (ellipsoidal versus astronomic) and the vertical (affected by gravity) versus normal (normal to ellipsoid, mathematical quantity) reference. The geodetic azimuth is what is determined using GPS, but is not directly measurable using any other common survey equipment. However, if one were to sight another survey station, and compute the geodetic inverse, then the resulting azimuth is a geodetic azimuth. Similarly, this is the type of azimuth required in the "direct" geodetic problem, where it is desired to compute the coordinates of P_2, given the coordinates of P_1, the geodetic azimuth, and the ellipsoidal distance between the two. Although the difference between the astronomic and geodetic azimuths is usually small, it is important to understand the difference, and know how to transform one to the other when needed.

The grid azimuth is defined as the angle measured in the horizontal plane between the grid meridian (which is parallel to the central meridian of a plane rectangular coordinate system, for example state plane or UTM) of P_1 and the vertical plane spanned by the normal to P_1 and P_2. This is very similar to the definition of the geodetic azimuth. It is possible to convert between geodetic azimuth and grid azimuth by applying the mapping angle (also known as convergence) and possibly the T-t correction (also known as the second term correction or the curvature correction). The T-t correction is only significant for long lines.

AZIMUTHS BY GPS

The Global Positioning System (GPS) can be (and often is) used to establish the azimuth between two points. Code GPS (differentially corrected) gives sub-meter accuracy with proper equipment. Differential Code GPS results in two positions, rather than a vector between the two points. Suppose it is desired to establish a pair of points separated by 500 m, and assume the accuracy of the processed point positions as 0.5 m. The azimuth would then have an uncertainty of almost 4 minutes of arc. However, if carrier phase observations are made, the resulting vector (ΔX, ΔY, ΔZ) would have an accuracy of about 3 seconds of arc. This is true even if the stations are not connected to a control network. However, if, for example, a NAD 1927 azimuth is needed, then it is important to use at least two control stations, since there can be local distortions of up to 10" or more between WGS84 and NAD27. Care must also be taken when using a non geocentric (i.e. local) datum, as the difference in longitude between WGS84 and the local system directly affects the azimuth, and can be computationally corrected.

GYROTHEODOLITE

Several manufacturers make a gyro attachment for a theodolite. The details of its operation can be found in the manufacturers manuals. The gyroscope oscillates about the astronomic meridian through a point, and the location of this meridian can be determined to an accuracy of 20" or better. These instruments are often used in tunneling and mine surveying. The high cost ($25,000 in the mid-80's) made their common use hard to justify. However, it is now possible to purchase used units for about $4500 (including theodolite).

These instruments are ideal for the short eccentric lines mentioned above, as the accuracy is sufficient, and they work independent of weather.

AZIMUTH MARKS & INTERSECTION STATIONS

First order triangulation stations were usually set in the US with an azimuth mark, which was simply another mark visible from the ground at least ¼ mile distance. Whenever a prominent landmark object (church steeple, radio tower, etc) was visible from several main scheme stations, observations were also made to them. The azimuth marks and intersection stations are considered third order, and provide azimuth orientation to about 5" to 10". One of the main problems with using intersection stations is determining if the correct point is being used, and, if so, whether it is in the same position as during the original observations. For example, church steeples are replaced, antennas are added to and modified, etc. In no case should nearby reference marks be used for azimuth control, as that was not the intent when they were established. An examination of triangulation descriptions will find situations where the station was revisited 30 years after the original observations, and discrepancies of a minute or more are found to reference marks.

ASTRONOMIC AZIMUTHS

In the past, solar observations were done by the azimuth-elevation method, where both the horizontal and vertical angles to the sun were needed. This method was used because it did not require accurate time, but was limited in accuracy by the refraction effect on the vertical angle. This method had a useful accuracy of about 1', and was therefore not used in geodetic control surveys. Now that accurate time ($20 stopwatch) and position ($150 handheld GPS receiver) are easily obtained, the method known as the hour angle method is commonly used.

Due to the simplicity and robustness of the hour angle method, that is the only method to be discussed here. Based on the availability of precise ephemerides, accurate timepieces, accurate observer positions, there is no compelling reason to require Polaris for azimuth observations of 5" accuracy or less. In fact, any object in space, with a good ephemerides, can be used. For accurate observations (better than 5"), it is still recommended that Polaris be used.

TIME

Time is the key to an accurate determination of astronomical azimuth. There are many types of time, and several important ones will be discussed here. International Atomic Time (TAI) is a continuous scale resulting from analyses of atomic time standards in many cooperating countries. The fundamental unit is the SI second, which is defined as the duration of 9,192,631,770 cycles of radiation corresponding to the transition between two hyperfine levels of the ground state of Cesium 133. Coordinated Universal Time (UTC) is the time scale available from broadcast time signals, and differs from TAI by an integer number of seconds (+32 seconds since Jan 1, 1999). It is kept close (within ±0.9 seconds) of the actual rotation of the earth (see UT1, below). Local time is really UTC offset by a integer number of hours (here in the US eastern time zone, -4 during daylight savings time, -5 otherwise). UTC is a uniform time scale kept by atomic clocks around the world, based on a mean solar time at the mean Greenwich meridian (0° longitude), and is corrected by leap seconds periodically (always on June 30 or December 31, if needed). These leap seconds are added (or subtracted) due to the non-uniform rotation of the earth. When the earth becomes out of synchronization with UTC by more than 0.5 seconds, a leap second is applied. UT1 is a measure of the actual rotation of the earth. The difference between UT1 and UTC is known as DUT (+0.07594 seconds for today, 2/8/01, with an accuracy of 0.0017 seconds). Celestial observations should be based on UT1, but since UT1 is not available from any clock, UTC corrected by DUT is used. GPS time is equivalent to UTC at January 5, 1980, uncorrected by leap seconds. Therefore, GPS time is currently offset from UTC by +13 seconds. Finally, Terrestrial Dynamic Time (TDT) is defined as the independent argument for apparent geocentric ephemerides. It is offset from TAI by -32.184 seconds, and is currently offset from UTC by -64.184 seconds.

Accurate UTC can be obtained from several sources. One of the most common is radio station WWV, in Fort Collins, Co. This station broadcasts on shortwave frequencies of 2.5, 5, 10, 15, and 20 MHz. Sister station WWVH in Hawaii broadcasts on the same frequencies. The two stations can be identified by a man's voice on WWV and a woman's voice on WWVH. Canada broadcasts on station CHU on 3.33, 7.335, and 14.67 MHz, and many other countries have similar services. Inexpensive time receivers which receive 5, 10, and 15 MHz are available from electronics stores for under $50. The same information can be obtained by telephone by calling WWV at (303) 499-7111. At 8 seconds before the minute, a voice gives the UTC time for the upcoming minute, which is indicated by a longer tick. After the minute tick, it is important to count the number of double ticks. Each double tick represents 0.1 s of DUT, and the sign of the correction is positive if given during seconds 1 to 7, and negative if given during seconds 9 to 15. The correction will never be greater than 0.7 seconds, and is presently +0.2.

$$UT1 = UTC + DUT\,1$$

Although this correction is usually small, it is easy to apply, and should be considered, especially for solar azimuths and stars other than Polaris. An easy way to measure time is as follows: Using a stopwatch with a lap function, start the stopwatch at the start of a minute. Record the minute and the DUT correction. It is a good idea to check the stopwatch after the first minute, and also to check the stopwatch at the end of the observations. Then, the watch is "lapped" (the display is stopped, but the watch keeps running) for each pointing on the celestial object. Time should be recorded to at least 0.1 s.

A final note about time. Values for α (right ascension) are usually given in units of time. This can be converted to degrees by multiplying by 15. A degree of longitude is equal to 4 minutes of time, and a second of longitude is equal to 0.067 seconds of time. Computations on a computer should be converted from units of time to degrees, then to radians.

POSITION

It is necessary to know the coordinates of the observation site. Latitude and longitude must be used to reduce the observations. These values can be scaled off of a quad, converted from state plane coordinates, or obtained from GPS. It should be mentioned that what are needed are astronomic coordinates, while geodetic coordinates are what we use in surveying. The National Geodetic Survey has a program (DEFLEC96, available from ftp.ngs.noaa.gov)) to compute the deflections of the vertical in the US, given a NAD 1983 latitude and longitude. Coordinates from a different datum, for example NAD 1927, should first be converted to NAD 1983. The output from the program is actually the deflections of the vertical in the meridian (ξ, xi) and the Prime Vertical (η, eta), so the astronomic latitude (Φ) and longitude (Λ) are computed from the geodetic values (ϕ, λ) as follows:

$$\Lambda = \frac{\eta}{\cos\Phi} + \lambda \qquad \Phi = \xi + \phi$$

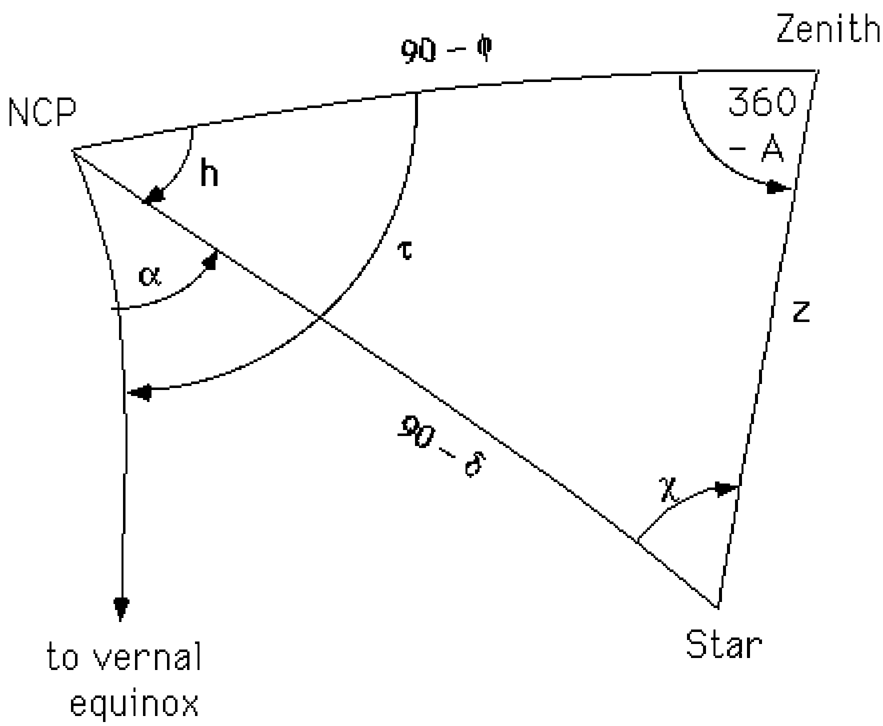
The sign convention for DEFLEC96 is such that a positive ξ means that $\Phi>\phi$, and a positive η means that $\Lambda<\lambda$, for west longitudes. Also output from this program is the Laplace correction, which is used to convert the astronomic azimuth to a geodetic azimuth, as explained later. It is important to note that the difference between astronomic and geodetic coordinates is small, often less than the observation noise. However, the values of η and ξ can become significant, and a knowledge of the local geoid is important, as these values represent the slope of the geoid. In an area with relatively little slope in the geoid, these values will be close to zero. Here are a few example values:

Area	η	ξ	Laplace	Comment
Uniontown, PA	-6.6″	5.1	5.5″	Allegheny Mtns
Cincinnati	2.8″	-1.8″	-2.3″	
Cheyenne, WY	14.9″	-3.2″	-13.8″	large E-W geoid slope
Meades Ranch (KS)	3.6″	-0.8″	-3.0″	origin for NAD27

In order to see the effect of errors in latitude and longitude (longitude = time), the azimuth equation is differentiated. The effect of an error in latitude is minimized when the azimuth to the body is 0° or 180°, while the error in longitude (and time) is minimized when the object is at elongation (i.e parallactic angle χ in diagram above is 90°). Of course, these two conditions cannot be met simultaneously. This is the reason why Polaris was used for so many years. It has a declination of about 89°, so its path around the pole has a radius of about 1°. In the past, when time was difficult to measure accurately in the field, Polaris was used at elongation, at which time it appeared to be moving vertically, and an error in time of several seconds or more could be tolerated, as well as an error in latitude. Today, this is less critical, since time can easily be measured to tenths of a second with an inexpensive stopwatch, and the astronomic station position can be computed as described above (at least in the US) to an accuracy of about 1" to 3". For other areas of the world, a method known as Black's Azimuth could be used. This method, proposed by A.N. Black, consists of azimuth observations on low altitude stars distributed around the horizon. By doing this, and using least squares, the geodetic azimuth can be computed directly, with estimates for η and ξ. In fact, even if an incorrect latitude and longitude are used, this method will give accurate results, since the deflections are inseparable from the position errors.

ASTRONOMICAL TRIANGLE

The key to understanding astronomic azimuths is the astronomical triangle. The celestial sphere is a unit sphere with the earth at the center, considered as a point. The stars are so far away they can be considered as points located at infinite radius. An hour circle is similar to terrestrial longitude, being a great circle perpendicular to the celestial equator and containing the celestial poles. The positions of stars on the sphere can be determined by two angular coordinates. There are various celestial coordinate systems, of which three will be considered here. The equatorial system is similar to the latitude-longitude system we use on earth. Right ascension (α) is the angle measured eastward along the equator from the vernal equinox, which is where the sun crosses the equator going from the southern hemisphere to the northern hemisphere (the instant spring starts) to the hour circle of the star. The declination (δ) is similar to latitude, in that it is the angular measure from the equator north or south along the hour circle to the celestial object. The hour angle system is simply the equatorial system rotated about the polar axis by the sidereal time (Θ), which is the angular difference along the equator between the vernal equinox and the local meridian. This then gives the hour angle (h) of the star. The declination is the same in both systems. The hour angle of a star is therefore the angle measured in the equatorial plane between the celestial meridian and the hour circle of the star. The last coordinate system to be considered here is the horizon system. This consists of the zenith angle of the star (90°-altitude) and the astronomical azimuth of the star. Clearly, this system varies with time and with position. The three system are related by simple rotational translations. The positions of stars ands other bodies are usually given in the equatorial system. Determining the azimuth of a celestial object is simply taking the observers position (ϕ and λ) and the time of observation to transform the equatorial coordinates of the object into the horizon coordinates. By examining the astronomical triangle, this process can be seen more clearly. The astronomical triangle is formed by the star (S), the zenith point (Z), directly overhead, where the plumbline intersects the celestial sphere, and the north celestial pole (NCP). The angle at each corner of the triangle is as follows: at the star, χ, also called the parallactic angle; at the zenith, A, the azimuth of the star; and at the pole, h, the hour angle of the star. The sides of the triangle are also expressed as angles, and are as follows: from NCP to S is the co-declination, $90°-\delta$; from NCP to Z is the co-latitude, 90°-latitude; and from Z to S is the zenith angle of the star.



In the figure above, the α is the right ascension and δ is the declination of the object (obtained from an ephemeris) for a given time t, ϕ is the latitude, A is the azimuth to the object, and τ is the Local Apparent Sidereal Time (LAST). The LAST is the Greenwich Mean Sidereal Time (GMST, obtained from an ephemeris), plus the equation of equinoxes, also from the ephemeris), plus the longitude λ (west negative). The hour angle of the object, h, is simply LAST - α, as shown above. Any unknown component of a triangle (spherical or plane) can be computed if other components are known. For example, the (obselete) altitude method of observation had the following known quantities: δ, Z, and ϕ (i.e. the three sides). The hour angle method has as known values two sides (ϕ and δ) and an angle, h. Given these values, the azimuth A can be computed:

$$TanA = \frac{\sin(h)}{\cos\Phi\tan\delta - \sin\Phi\cos(h)}$$

The azimuth is clockwise from north, but must be corrected as follows:

000° < h < 180° and A is positive: add 180°
000° < h < 180° and A is negative: add 360°
180° < h < 360° and A is positive: accept A
180° < h < 360° and A is negative: add 180°

Once the azimuth A is known, the observed horizontal angle at the standpoint from the celestial object to the forepoint is used to compute the azimuth to the forepoint.

INCLINATION OF STANDING AXIS

The largest error source when using the hour angle method is the inclination of the standing axis of the theodolite. This error source is frequently overlooked, and *CANNOT* be eliminated by observing direct and reverse. The error dH is as follows:

$$dH = m * \tan V$$

where m is the misleveling, in seconds of arc, and V is the vertical angle. A Wild T-2, for example, has a plate level which has a sensitivity of about 20"/2mm division. This means that if the bubble is off center (assuming center is level) by 1.5 divisions, and the altitude of the sun is 40°, then the azimuth will be in error by 25", no matter how many sets are observed. Using the newer theodolites, with automatic compensator for the vertical circle, it is possible to either measure this inclination accurately, or to greatly reduce it, by the following procedure. When leveling up, rotate the instrument so that the telescope is directly pointed over one foot screw. Lock the vertical motion, and read the vertical circle. Rotate the instrument (do not flop) 180°, and read again. Take the difference, and divide by 2. This is the amount the instrument is out of level perpendicular to the direction the telescope is pointing. Take the mean of the two readings, set it on the micrometer, and use the two level screws on the line perpendicular to the telescope direction to bring the micrometer into coincidence. Rotate the telescope 90°, and repeat the procedure. After several iterations, there will be no change in any direction, and the theodolite is level. However, it is difficult to get the theodolite perfectly level. Usually, the reference mark is close to the same horizontal plane as the station (i.e. the vertical angle is close to 0°), so there is no effect on that direction. It is possible to accurately measure the inclination of the standing axis perpendicular to the direction to the sun. Before and after each set, turn the telescope 90° from the sun. Tighten the vertical clamp. Read the vertical circle. Rotate 180°, and repeat. The correction is as follows:

$$C = \frac{V_R - V_L}{2}\tan\beta$$

where V_R and V_L are the vertical circle readings right and left, respectively, and β is the vertical angle to the sun (computed). Note that it doesn't matter what vertical angle is used left and right, only that the telescope be clamped. By carefully doing this, it is possible to measure the inclination to an accuracy of several seconds, and correct the azimuth by subtracting the correction (paying attention to the sign of the correction) from the computed azimuth. Many of the more advanced total stations have dual axis compensators, which can measure and automatically correct this error.

Of course, this error can be greatly minimized by observing objects close to the horizon. For example, if the mislevelment is 10”, and the sun is at 10°, the error introduced is 1.8”. However, if the sun is at 45°, the same leveling error will cause an error of 10” in the azimuth.

LAPLACE CORRECTION

An azimuth measured with a theodolite is an astronomic azimuth. It refers to an azimuth expressed in a local horizon coordinate system aligned along the local gravity vector (line of the plumb bob). A geodetic azimuth, on the other hand, is expressed about a local normal to the ellipsoid. An astronomic azimuth is converted into a geodetic azimuth by applying the Laplace correction. The Laplace correction typically takes one of two forms:

The simplified Laplace Correction (horizontal Laplace):

$\eta * \tan\phi$ 2

The complete (Extended) Laplace Correction:

$\eta * \tan\phi - (\xi * \sin(AZ) - \eta * \cos(AZ)) * \cot(ZD)$ 3

The second term is known as the deflection correction, and it is negligible for horizontal lines of sight (ZD is the zenith angle to the reference mark, AZ is the azimuth to the reference mark). The computed Laplace correction (horizontal Laplace) should be ADDED to a clockwise astronomic azimuth, to obtain a "near-geodetic" Laplace azimuth.

OBSERVATION PROCEDURE

In order to determine an astronomic azimuth using the hour angle method, the following additional equipment is required:

1) stopwatch - $20
2) eyepiece filter for optical theodolite (not absolutely necessary, can use projection method)

or

3) objective filter for total station (must have)

In order to perform the computations, the following values are required:

1) Angle from mark to astronomic object (observed)
2) time of pointing (observed)
3) position of object (computed)
4) position of observer (from map, GPS, or given control)
5) diameter of sun or moon, n/a if using a star (computed)

The basic observation sequence is as follows:

1) start clock, record UTC start time and DUT
2) point and record backsight
3) point to star or trailing edge of sun, press lap button on watch at the instant of passage
4) record time and horizontal circle reading

5) invert telescope, repeat steps 3 and 4
6) point and record backsight

Steps 2 through 6 constitute one set. When using the sun, it is advisable to do multiple foresights, for example 3D and 3R in each set. The procedure would be 1-2-3-4-3-4-3-4-5-3-4-3-4-3-4-6. At the end of all the sets (usually an even number), step 1 is repeated. Proper leveling of the instrument is the most critical and neglected operation when observing azimuths. It cannot be emphasized enough the importance of proper leveling of the instrument. A good instrument operator can quickly become proficient at observing the sun or stars, and marking the correct time, but if the instrument is not level, the observational results will be poor.

SOLAR OBSERVATIONS

When using the Sun, the more difficult sighting is at the Sun, rather than the ground forepoint (assumed to be a target). Since the computations give the azimuth to the *center* of the sun, there several options. A Roelofs Prism can be used (made for a Wild T-2). This device fits over the objective end of the telescope, and gives four overlapping images of the sun, which form a cross in the center against which the crosshairs can be seen. More feasible is to observe an edge of the sun, and correct the angle by the semi-diameter of the sun. This is defined as the angle subtended by the equatorial radius of the sun. The semi-diameter can be obtained from an ephemeris as well. Note that the correction to the sun's azimuth is semi-diameter * sec (altitude). The altitude can be computed from:

$$altitude = \arcsin\left(\sin\Phi\sin\delta + \cos\Phi\cos\delta\cos(h)\right)$$

It is much easier to observe the trailing edge of the sun. Note that in latitudes between about -23°30' and +23°30' (i.e. ± max δ), this can be either the left edge or the right edge, depending on time of year, whereas above +23°30, it is always the left edge. The sign of the semi-diameter correction depends on whether the left or right edge is observed.

There are two methods for observing the sun. If a conventional theodolite is used (i.e. no coaxial EDM), then an image of the sun can be projected onto a white card or field book page held behind the eyepiece. DO NOT DIRECTLY OBSERVE THE SUN THROUGH THE EYEPIECE WITHOUT A FILTER. An inexpensive eyepiece filter can also be used, which is nothing more than a piece of smoked glass. If an EDM is present, it is necessary to use a filter on the objective end, to keep from burning out the optics in the EDM. Because this filter is before the lenses in the instrument, and the filters are not precision optical lenses, an error will be introduced into the observations, but this error will be eliminated by observing both direct and inverted.

In order to assess the accuracy of a single set for a solar observation, the following assumptions are made:

1) time is accurate to 0.2s (easily obtained with modern stopwatches, must be corrected for DUT1
2) latitude and longitude errors (astronomic) are less than 2"

3) instrument is within 5" of level (see discussion of inclination error above), or the inclination is measured to an accuracy of better than 5"
4) 1" instrument

Based on this, an estimate of 5" is obtained. By observing n sets, the accuracy is improved as follows:

$$\sigma_n = \frac{\sigma}{\sqrt{n}}$$

For a less accurate set of circumstances, assume a 6" theodolite, position errors of 5", instrument level to within 30", and time accurate to 1s. This gives an accuracy of a single set as about 25", certainly accurate enough for a short eccentric reduction.

POLARIS OBSERVATIONS

Using Polaris has several advantages over using the sun. The effects of small errors in time and position are greatly reduced, almost negligible. The following table, taken from the 1992 Sokkia Celestial Observation Handbook and Ephemeris, show the maximum error in azimuth related to errors in time, latitude, and longitude:

Error Type	ϕ=20°	ϕ=40°	ϕ=60°
1s of time	0.23″	0.28″	0.42″
10″ λ	0.05″	0.15″	0.49″
10″ ϕ	0.15″	0.18″	0.28″

The star appears as a point of light, easy to point to with the crosshair. The disadvantages are that observations must be made at night, although on a clear day, with a properly focused instrument, it is possible (though difficult) to find and use polaris through the telescope. The other disadvantage is that the altitude of polaris is approximately equal to the latitude. Therefore, in mid-latitudes, any inclination error has a significant effect on the computed azimuth, if not measured and corrected.

The procedures and computations are basically the same as for the sun, except for the semi-diameter correction.

SOLAR EXAMPLE

There are various software programs available for reducing astronomic observations. An example computation for the sun is given here to enhance the users understanding of the process. The Astronomical Almanac 1997 was used , and is available from the Government Printing Office (GPO) for $35.

```
Station = SUMMIT (PID=JW1336)
10/14/97 15:22:39.65 EDT      UTC=19:22:39.65   ΔUT=+0.36s
ϕ=39°51′02.51677″ Nλ=79°39′24.44579″W
(observed Φ=39°51′07.0″ N            Λ=79°39′33.93″W)
angle from RM to trailing edge of sun: 223°47′19″
vertical angle lt=67°12′23″    vertical angle rt=67°12′25″
1) compute astro coords (assume they were not observed) from DEFLEC96:
      ξ=5.14″      η= -6.62″   Laplace=+5.53″
   Φ=39°51′07.7″ N       Λ=79°39′31.1″W
2) compute UT1 from UTC and ΔUT     UT1=19:22:40.01
```

```
3) compute LAST
      GMST at 0h                        01:30:18.9781
      ΔMST from 0h to 19:22:40:01       19:25:51.0066
      GMST at 19:22:40.01                     20:56:09.9847
      equation of equinoxes                   -0.2921
      GAST at 19:22:40.01                     20:56:09.6926
      subtract west longitude           05:18:38.0733
      LAST                              15:37:31.6192
4) compute TDT=UTC+63.18      TDT=19:23:42.83
5) interpolate α, δ and SD for TDT:
      α at 0h TDT:                            13:16:24.78
      α at 24h TDT                            13:20:07.46
      α at 19:23:42.83                  13:19:24.74

      δ at 0h TDT                       -8°04′30.2″
      δ at 24h TDT                      -8°26′46.6″
      δ at 19:23:42.83                  -8°22′30.2″

      SD at 0h TDT                             0°16′02.11″
      SD at 24h TDT                            0°16′02.39″
      SD at 19:23:42.83                  0°16′02.34″
6) compute local hour angle (LAST-α):     02:18:06.88
convert to degrees (multiply by 15) 34°31′43.19″
7) compute azimuth
            A=221°29′15.8″
8) compute and apply semi-diameter correction:
      altitude=32°10′00″
      correction = SD*sec(altitude) = 0°18′56.8″
      azimuth to trailing edge=221°10′19.0″
9) compute inclination correction (<1″, not applied)
10)subtract angle from RM to obtain astronomic azimuth to RM
      357°23′00″
11) apply Laplace Correction to obtain geodetic azimuth
      357°23′05.5″
```

LUNAR EXAMPLE

In fact, any celestial object with an accurate ephemeris can be used. An often overlooked possibility is a lunar azimuth determination. Frequently, the moon is visible in daylight hours. The advantage here is that no filter is necessary. However, there are several differences in the computations. Polynomial coefficients must be used to compute the α and δ at the time of observation. Because the radius of the earth is not infinitesimally small compared to the distance to the moon, the geocentric α and δ must be converted to topocentric values. For example:

```
Station = CLIFTON
08/07/97 20:18:05.30 EDT      UTC=00:18:05.30 (8/8/97)      ΔUT=+0.48s
φ=40°17′54.23″ N   λ=80°02′42.19″W    ellip h=267 m
angle from R to leading edge of moon: 20°09′32″
vertical circle left=278°36′10″      right=278°34′54″
1) compute astro coords from DEFLEC96: ξ= 2.54″ η=                -2.53″
   Laplace=+2.15″
   Φ=40°17′56.8″ N       Λ=80°02′44.7″W
2) compute UT1 from UTC and ΔUT     UT1=00:18:05.78
3) compute LAST
      GMST at 0h                    21:06:09.7685
      ΔMST from 0h to 00:18:05:78   00:18:08.7528
      GMST at 00:18:05.78                 21:24:18.5213
      equation of equinoxes               -0.0886
      GAST at 00:18:05.78                 21:24:18.4327
      subtract west longitude       05:20:10.9800
      LAST                          16:04:07.4527
4) compute TDT=UTC+63.18      TDT=00:19:08.48
5) using the polynomial coefficients (page D40 of the 1997 Astronomical
   Almanac), compute α, δ, and HP (horizontal parallax) for TDT:
      α=12:25:48.826          δ=-1°11′10.61″    HP=0°54′08.950″
6) convert latitude, longitude, ellipsoidal height of station to ECEF
   XYZ:
      X=(N+h)cosφ * cosλ
      Y=(N+h)cosφ * sinλ
      Z=(1-e²)N+h)sinφ
      where N is the radius of curvature in the prime vertical:
```

$$N = \frac{a^2}{\sqrt{a^2 \cos \ \phi + b^2 \sin^2 \phi}};$$

```
      a is the semimajor axis and b is the semiminor axis of the
      earth ellipsoid, a=6,378,137 m, b=6,356,752.314 m

      X=              842,163 m
      Y=           -4,798,202 m
      Z=            4,103,484 m
```

7) compute the geocentric distance of the station, $\rho = \sqrt{X^2 + Y^2 + Z^2}$

ρ=6,369,502 m

and the geocentric latitude of the station: $\tan\psi = \frac{b^2}{a^2}\tan\phi$

ψ=40°06′31.2″

8) compute the true distance to the moon:

$$r = \frac{6378137m}{\sin(HP)}$$ = 404,943,011 m

9) compute the topocentric position vector of the moon:

(θ0 is the LAST)

$X' = r\cos\delta\cos\alpha - \rho\cos\psi\cos\theta_0$ = -399,931,391 m

$Y' = r\cos\delta\sin\alpha - \rho\cos\psi\sin\theta_0$ = - 41,242,112 m

$Z' = r\sin\delta - \rho\sin\psi$ = - 12,487,028 m

10) compute topocentric values:

$r' = \sqrt{X'^2 + Y'^2 + Z'^2}$ = 402,246,137 m

$\alpha' = \tan^{-1}\left(\frac{Y'}{X'}\right)$ = 12:23:33.0478

$\delta' = \sin^{-1}\left(\frac{Z'}{R'}\right)$ = -1°46′44.16″

11) compute the topocentric hour angle:

$h' = \theta_0 - \alpha'$ = 55°08′36.073″

12) compute the azimuth to the center of the moon:

A=244°23′24.5″

13) compute and apply the semi-diameter correction

altitude=24°33′23

$SD = \sin^{-1}(0.272493 * \sin HP)$ = 0°14′45.3″

semi-diameter correction = 0°16′13.3″

azimuth to leading edge =244°39′37.8″

14) correct the azimuth for inclination of standing axis:

C=-17.4″

corrected azimuth=244°39′55.2″

15) subtract angle from RM to obtain astro azimuth to RM:

astro azimuth to RM=224°30′23″

16) apply Laplace Correction to obtain geodetic azimuth:

geodetic azimuth to RM=224°30′25.3″

These steps, while tedious, could be easily programmed into a computer.

PROPOSED SPECIFICATIONS FOR SHORT LINES

As mentioned earlier in the paper, the accuracy required for a short eccentric observation is much less than that required to extend control via traverse or triangulation. The following would apply for lines up to 50 meters in length.

- A 1 second instrument should be used, with either an automatic compensator or dual axis compensation. If an automatic compensator is present (Wild T-2, Kern DKM 2, for example), then the instrument should be leveled up using the procedure outlined above, and any residual inclination should be measured and recorded before and after each set. If dual axis compensation is present then the function should be tested according to the manufacturers instructions at the start and end of the observations.
- When using the sun, observations should not be made within two hours of local noon
- time should be observed and recorded such that an accuracy of 0.2s is attained

- computations should be made using the astronomic positions, and the Laplace correction applied
- at least two sets should be observed, separated by at least 1 hour. The mean of the sets should not differ by more than 5”

SUMMARY

Short eccentric lines do not require high accuracy azimuth determinations. The simple methods and computations for obtaining azimuths by solar means was presented. It is hoped that the FGCS will work to incorporate some form of this method for eccentric reductions, such as found in GPS survey networks, without the cumbersome requirement for polaris observations.